Ch2: Equations & Inequalities

Remark: In this chapter we will continue what we began in section 1.3. We will continue discussing the quadratic formula as well as proving it! **You will be required to prove the quadratic formula on the first exam.**

Although the quadratic formula essentially solves all quadratic equations, in many cases it is still convenient to solve by factoring. **It is essential that you are confident in factoring to pass this course!**

I. The Quadratic Formula 2.1

Ex: Solve by square root method.

$$(2x - 3)^2 = 28$$

Perfect square:

Recall $(A + B)^2 = A^2 + 2AB + B^2$

We can re-write as $(x + b)^2 = x^2 + 2bx + b^2$

Ex: Solve by completing the square.

$3x^2 + 2x - 3 = 0$

Ex: Prove the quadratic formula.

$ax^2 + bx + c = 0$

Ex: Use quadratic formula to solve.

1) $3x^2 + 2x - 3 = 0$

2) $x^2 - 2x = 8$

3) $x^2 + 2x + 7 = 0$

4) $-\frac{1}{2}gt^2 + v_0 t + h_0 = 0;\ \textit{for } t$

II. The Discriminant

Discuss the graphs of the following and their **solutions**.

$y = 2x^2 + x - 15$ $y = x^2 + 2x + 15$ $y = x^2 + 4x + 4$

Def: Given $ax^2 + bx + c = 0$,
where a, b, & c are real with $a \neq 0$.
The expresion

$$b^2 - 4ac$$

is called the **discriminant**.

Quadratic Eqn:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant can be used to determine the number and type of solutions to a quadratic equation.

1) $b^2 - 4ac > 0$ ⟹ two distinct real roots.

2) $b^2 - 4ac < 0$ ⟹ exactly one distinct root.

3) $b^2 - 4ac < 0$ ⟹ No real roots (two complex conjugate roots)

Ex: Use the determinant to determine how many real solutions each equation has.

1) $2x^2 + x - 15 = 0$

3) $x^2 + 4x + 4 = 0$

2) $x^2 + 2x + 15 = 0$

Ex: Determine the value(s) of the constant k for which the equation has exactly one distinct (or repeated) root.

$$x^2 + 2(k + 1)x + k^2 = 0$$

III. More Equations 2.2

A. Absolute Value Equations

a is pos number,

$|u| = a \Rightarrow u = a$ or $u = -a$

Exercises

Solve

1) $2x - 3 = 7$

2) $|2x - 3| = 7$

3) $|2x - 3| = -7$

B. n[th] Root Method

Recall:

$$\sqrt{x^2} =$$

$$\sqrt[3]{x^3} =$$

$$\sqrt[4]{x^4} =$$

$$\sqrt[5]{x^5} =$$

$$\sqrt[6]{x^6} =$$

$$\vdots$$

Ex: Solve

1) $(4x-1)^3 = -16$

2) $(2x+5)^4 = 81$

C. By Factoring

1) $2x^3 - 5x^2 + 3x = 0$

2) $x^4 - 7x^2 + 12 = 0$

D. Quadratic Type

Q: Why are the following quadratic types?

1) $x^{2/3} + 2x^{1/3} - 15 = 0$

2) $6x^{-2} - 5x^{-1} - 6 = 0$

3) $x^4 - 6x^2 + 4 = 0$

Ex: Solve

1) $x^{2/3} + 2x^{1/3} - 15 = 0$

2) $6x^{-2} - 5x^{-1} - 6 = 0$

3) $x^4 - 6x^2 + 4 = 0$

E. Radical Equations

1) $\sqrt{2x+7} - 2 = x$

2) $\sqrt{2x+3} = 2 + \sqrt{x-2}$

IV. Linear Inequalities 2.3

Rules for Inequalities

- $a \le b \Rightarrow a + c \le b + c$
- $a \le b$ & c is positive $\Rightarrow ca \le cb$
- $a \le b$ & c is negative $\Rightarrow ca \ge cb$

Ex: Solve. Write answers in interval notation.

1) $-6(x - 2) > 2 - 4x + 5(x - 1)$ **2)** $-1 < 6y + 5 \le 4$

Ex: Section 2.3, example 3

The formula relating temperature between Fahrenheit (F) & Celsius (C) is given by

$$F = \frac{9}{5}C + 32.$$

Over the temperature range $32^\circ \le F \le 39.2^\circ$ on the Fahrenheit scale, water contracts with increasing temperature. What is the corresponding temperature range on the Celsius scale?

V. Absolute Value Inequalities

Given a > 0,

$|u| < a$ iff $-a < u < a$

$|u| > a$ iff $u > a$ or $u < -a$

Ex: Solve. Graph and write answer in interval notation.

1) $|2x - 5| + 3 < 16$

2) $|3x + 4| - 7 \geq 18$

3) $|5x - 4| < -10$

4) $|5x - 4| > -10$

Overview

Ex: Solve the following.

1) $\frac{2x+3}{5} = 5$

2) $\left|\frac{2x+3}{5}\right| = 5$

3) $\left|\frac{2x+3}{5}\right| \leq 5$

4) $\left|\frac{2x+3}{5}\right| \geq 5$

5) $\frac{2x+3}{5} \geq 5$

VI. Nonlinear Inequalities 2.4

Consider $x^2 - 2x - 15 < 0$

Graphical Approach

Look at the graph of $y = x^2 - 2x - 15$

Algebraic Approach

Idea: Consider the possibilities for each of the inequalities.

PQ < 0 PQ > 0

Ex: Use a "sign chart" to solve

$x^2 - 2x - 15 < 0$

Use the sign chart on the previous problem to find solutions to the following:

$x^2 - 2x - 15 \leq 0$

$\frac{x+3}{x-5} > 0$

$x^2 - 2x - 15 > 0$

$\frac{x+3}{x-5} \geq 0$

$x^2 - 2x - 15 \geq 0$

$\frac{x-5}{x+3} < 0$

Ex: Solve. Write answer in interval notation.

$$x^3 - 8x^2 + 12x > 0$$

Ex: Solve. Write answer in interval notation.

$$\frac{4}{x-2} \leq \frac{3}{x-1}$$