Precalculus Chapter 1 Quiz Part I - ***Solutions to most-missed problems.***

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3. $\left[-6, -\frac{1}{4}\right] = \boxed{\left\{x \,\middle|\, -6 \le x \le -\frac{1}{4}\right\}}$

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4. The property that allows one to equate $9x\cdot(10+y) = (10+y)\cdot 9x$,
   is the commutative property of multiplication: $A\cdot B = B\cdot A$.

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9. $(4b)^{\frac{1}{2}}\left(7b^{\frac{2}{3}}\right) = 4^{\frac{1}{2}}\cdot b^{\frac{1}{2}}\cdot 7\cdot b^{\frac{2}{3}} = 2\cdot 7\cdot b^{\frac{1}{2}}\cdot b^{\frac{2}{3}} = 14\cdot b^{\frac{1}{2}+\frac{2}{3}} = \boxed{14\cdot b^{\frac{7}{6}}}$

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10. $\left(x^{-5}y^{8}z^{4}\right)^{-\frac{10}{3}} = x^{(-5)\left(-\frac{10}{3}\right)}y^{8\left(-\frac{10}{3}\right)}z^{4\left(-\frac{10}{3}\right)} = x^{\frac{50}{3}}y^{-\frac{80}{3}}z^{-\frac{40}{3}} = \boxed{\dfrac{x^{\frac{50}{3}}}{y^{\frac{80}{3}}z^{\frac{40}{3}}}}$

13.

$$\frac{(72.3)\left(1.3561\times10^{23}\right)}{0.000000027} = \frac{\left(7.23\times10^{1}\right)\left(1.3561\times10^{23}\right)}{2.7\times10^{-8}} = \frac{(7.23)(1.3561)}{2.7}\times10^{1+23-(-8)}$$

$$= 3.631334444\times10^{32}$$

The problem asks us to state our answer "correct to the number of significant digits indicated by the given data. Since the least number of significant digits of all the numbers used in the calculation is 2 (the number in the original denominator), we should round off our answer to 2 digits: $\cong \boxed{3.6\times10^{32}}$

(I gave credit on this one for either the above answer, or the less rounded one.)

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19. For this one, factor out the smallest power common to each of the two terms:

$$2x^{\frac{1}{3}}(x-2)^{\frac{2}{3}} - 7x^{\frac{4}{3}}(x-2)^{-\frac{1}{3}} = x^{\frac{1}{3}}(x-2)^{-\frac{1}{3}}\left(2(x-2)^{\frac{3}{3}} - 7x^{\frac{3}{3}}\right) = x^{\frac{1}{3}}(x-2)^{-\frac{1}{3}}\left(2(x-2)^{1} - 7x^{1}\right)$$

$$= x^{\frac{1}{3}}(x-2)^{-\frac{1}{3}}(2x-4-7x) = \boxed{x^{\frac{1}{3}}(x-2)^{-\frac{1}{3}}(-5x-4)} \text{ or } \boxed{-x^{\frac{1}{3}}(x-2)^{-\frac{1}{3}}(5x+4)}$$

22.

$$\frac{n^{-1}+m^{-1}}{(n+m)^{-7}}=\frac{n^{-1}+m^{-1}}{(n+m)^{-7}}\cdot\frac{nm(n+m)^{7}}{nm(n+m)^{7}}=\frac{(n^{-1}nm+m^{-1}nm)(n+m)^{7}}{(n+m)^{-7}\,nm(n+m)^{7}}=\frac{(m+n)(n+m)^{7}}{nm}=\boxed{\frac{(n+m)^{8}}{nm}}$$

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23. $$\frac{1}{\sqrt{7}+\sqrt{3}}=\frac{1}{\sqrt{7}+\sqrt{3}}\cdot\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}=\frac{\sqrt{7}-\sqrt{3}}{\left(\sqrt{7}\right)^{2}-\left(\sqrt{3}\right)^{2}}=\frac{\sqrt{7}-\sqrt{3}}{7-3}=\boxed{\frac{\sqrt{7}-\sqrt{3}}{4}}$$

26.

$$\frac{5}{x}-\frac{33}{x+6}+2=0 \quad\Rightarrow\quad \frac{5\cdot x\cdot(x+6)}{x}-\frac{33\cdot x\cdot(x+6)}{(x+6)}+2\cdot x\cdot(x+6)=0\cdot x\cdot(x+6)$$

$$\Rightarrow\quad 5\cdot(x+6)-33\cdot x+2\cdot x\cdot(x+6)=0$$

$$\Rightarrow\quad 5x+30-33x+2x^{2}+12x=0$$

$$\Rightarrow\quad 2x^{2}-16x+30=0$$

$$\Rightarrow\quad x^{2}-8x+15=0$$

$$\Rightarrow\quad (x-3)(x-5)=0 \quad\Rightarrow\quad \boxed{x=3 \text{ or } x=5}$$

Both of these possible solutions work when substituted into the original equation. (Remember, for this type of problem, we only need to check that these possible solutions don't make any of the original denominators 0.)

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30. Find all real solutions to the equation, $\sqrt{4x+16}+4=x$.

Our strategy with this type of problem is to get the radical all by itself on one side of the equal sign, and then square both sides:

$$\sqrt{4x+16}+4=x \quad \Rightarrow \quad \sqrt{4x+16}=x-4 \quad \Rightarrow \quad \left(\sqrt{4x+16}\right)^2=(x-4)^2$$

$$\Rightarrow \quad 4x+16=x^2-8x+16 \quad \Rightarrow \quad 0=x^2-12x \quad \Rightarrow \quad 0=x(x-12)$$

$$\Rightarrow \quad \boxed{x=0 \text{ or } x=12}$$

We need to check to see if these possible solution work in the original equation:

$$\boxed{x=0}: \quad \sqrt{4\cdot 0+16}+4\overset{?}{=}0 \quad \Leftrightarrow \quad \sqrt{16}+4\overset{?}{=}0 \quad \Leftrightarrow \quad 4+4\overset{?}{=}0 \quad \Leftrightarrow \quad 8\overset{?}{=}0 \quad \Leftrightarrow \quad \mathit{No}!$$

$$\boxed{x=12}: \quad \sqrt{4\cdot 12+16}+4\overset{?}{=}12 \quad \Leftrightarrow \quad \sqrt{48+16}+4\overset{?}{=}12 \quad \Leftrightarrow \quad 8+4\overset{?}{=}12 \quad \Leftrightarrow \quad \mathit{Yes}!$$

So our only real solution is $\boxed{x=12}$.
