Chapter 1 Quiz Part 2 - **Solutions to Most-Missed Problems**

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5. A merchant blends tea that sells for \$2.85 per pound with tea that sells for \$2.60 per pound to produce 90 lb of a mixture that sells for \$2.75 per pound. How many pounds of each type of tea does the merchant use in the blend?

Let C = pounds of cheap tea (the one that sells for \$2.60/lb), and
Let E = pounds of expensive tea (the one that sells for \$2.85/lb).

Then the info given in the problem suggests the linear system:

$$\begin{cases} \text{E} + \text{C} = 90 \\ 2.85\text{E} + 2.60\text{C} = 2.75(90) = 247.50 \end{cases}$$

One can solve these two equations with two unknowns in various ways (substitution, etc).

The addition method works like this (I noticed that multiplying the second equation by 20 would eliminate all the decimals; then I multiplied the top equation by –52 to cancel the C's):

$$\begin{cases} E + C = 90 \\ 2.85E + 2.60C = 247.50 \end{cases} \begin{matrix} \xrightarrow{\cdot(-52)} \\ \xrightarrow{\cdot 20} \end{matrix} \begin{cases} -52E - 52C = -4680 \\ 57E + 52C = 4950 \end{cases}$$

Adding these two equations: $5E = 270 \;\Rightarrow\; E = \dfrac{270}{5} = 54,$

so we need 54 pounds of the tea that sells for \$2.85 per pound,
and 90 – 54 = 36 pounds of the tea that sells for \$2.60 per pound.

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10. Solve $\dfrac{3}{x-1} - \dfrac{4}{x} \geq 1.$ Express the solution using interval notation and graph the solution set.

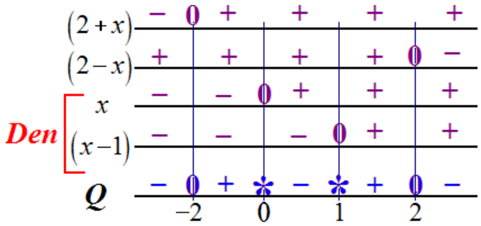
$$\frac{3}{x-1} - \frac{4}{x} - 1 \geq 0 \quad \Rightarrow \quad \frac{3x}{(x-1)x} - \frac{4(x-1)}{x(x-1)} - \frac{x(x-1)}{x(x-1)} \geq 0$$

$$\Rightarrow \quad \frac{3x - 4(x-1) - x(x-1)}{x(x-1)} \geq 0$$

$$\Rightarrow \quad \frac{3x - 4x + 4 - x^2 + x}{x(x-1)} \geq 0 \quad \Rightarrow \quad \frac{4 - x^2}{x(x-1)} \geq 0$$

$$\Rightarrow \quad \boxed{\frac{(2+x)(2-x)}{x(x-1)} \geq 0}$$

Next we make our little +– chart:



Notice that we are looking at where $Q = \dfrac{(2+x)(2-x)}{x(x-1)} \geq 0$, i.e., we are looking in the above chart for where Q is + and where Q is 0. Thus the solution set for our nonlinear inequality Q, is $[-2,0)\cup(1,2]$.

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13. Solve the inequality $5x+24<9$. Express the solution using interval notation.

$$5x+24<9 \quad \Rightarrow \quad 5x<-15 \quad \Rightarrow \quad x<-3$$

Thus the solution is all x in the interval $(-\infty,-3)$.

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14. In the vicinity of a bonfire, the temperature T in °C at a distance of x meters from the center of the fire was given by

$T = \dfrac{762500}{x^2+300}$. At what range of distances from the fire's center was the temperature less than 500°C?

We'll solve the inequality, $500 > \dfrac{762500}{x^2+300}$. Notice that the denominator is ALWAYS positive. So we can multiply both sides by this denominator, thus obtaining $500\left(x^2+300\right) > 762500$. Dividing both sides by 500 yields: $x^2+300>1525$, and then subtracting 300 from both sides results in $x^2>1225$. Taking the square root of both sides, realizing that x must be positive, we get: $x>35$.

Points more than 35 meters away from the fire will be less than 500°C.

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15. Solve the inequality $20-4x\leq-16$. Express the solution using interval notation.

$$20-4x\leq-16 \quad \Rightarrow \quad -4x\leq-36 \quad \Rightarrow \quad \frac{-4x}{-4}\geq\frac{-36}{-4} \quad \Rightarrow \quad x\geq 9$$

This solution set is the interval $[9,\infty)$.

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17. Solve the nonlinear inequality $\dfrac{x}{x+1} > 3x$. Express the solution using interval notation and graph the solution set.

$$\frac{x}{x+1} > 3x \quad \Rightarrow \quad \frac{x}{x+1} - 3x > 0 \quad \Rightarrow \quad \frac{x}{x+1} - \frac{3x(x+1)}{(x+1)} > 0 \quad \Rightarrow \quad \frac{x - 3x(x+1)}{x+1} > 0$$

$$\Rightarrow \quad \frac{x - 3x^2 - 3x}{x+1} > 0 \quad \Rightarrow \quad \frac{-3x^2 - 2x}{x+1} > 0 \quad \Rightarrow \quad \boxed{\frac{-x(3x+2)}{x+1}} > 0$$

Next we make our little +– chart:

	$x<-1$	-1		$-2/3$		0	$x>0$
$-x$	+		+		+	0	−
$(3x+2)$	−		−	0	+		+
Den $(x+1)$	−	0	+		+		+
Q	+	*	−	0	+	0	−

Notice that we are looking at where $Q = \dfrac{-x(3x+2)}{x+1} > 0$, i.e., we are looking in the above chart for

where Q is +. Thus the solution set for our nonlinear inequality Q, is $\boxed{(-\infty,-1)\cup\left(-\frac{2}{3},0\right)}$.

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18. Solve the inequality $5|x+2|+9>8$. Express the answer using interval notation.

$$5|x+2|+9>8 \quad \Rightarrow \quad 5|x+2|>-1$$

Notice that the left side of the equal sign is positive (or zero), which thus HAS to be larger than negative one. Thus, any real value for x will result in the inequality being true. The solution set is thus all real numbers, which in interval notation is written as $\boxed{(-\infty,\infty)}$

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19. Express the phrase "All real numbers *x* at least 3 units from 4 " as an inequality involving an absolute value.

We seek all real numbers *x* that are at a distance of 3 or more units from 4, or in other words, the distance from *x* to 4 is greater than or equal to 3: $\boxed{|x-4|\ge 3}$

(Remember that the distance from A to B is given by | A - B | .)

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20. Determine the values of the variable for which the expression $\left(\frac{1}{x^2-x-20}\right)^{\frac{1}{2}}$ is defined as a real number.

To be a real number, the denominator must be positive (that ½ power means the same as "square root"):

$$0 < x^2 - x - 20$$
$$0 < (x+4)(x-5) = \text{Product}$$

Making our little +- chart:

x + 4	**–**	**0**	**+**		**+**	
x – 5	**–**		**–**	**0**	**+**	
Product	**+**	**0**	**–**	**0**	**+**	
		–4		**5**		

So $\left(\frac{1}{x^2-x-20}\right)^{\frac{1}{2}}$ is defined as a real number whenever that product is positive, on the set:

$$\boxed{(-\infty,-4)\cup(5,\infty)}$$

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23. Find a point on the *y*-axis that is equidistant from the points (1, –7) and (5, –1).

Points on the *y*-axis are all of the form (0, *y*), and so we seek a y such that

The distance from (1, –7) to (0, *y*), is equal to the distance from (5, –1) to (0, *y*).
It's easier to square both sides to eliminate the square roots ;

$$(1-0)^2+(-7-y)^2=(5-0)^2+(-1-y)^2$$
$$1+49+14y+y^2=25+1+2y+y^2$$
$$50+14y=26+2y$$
$$12y=-24 \quad \Rightarrow \boxed{y=-2}$$

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26. Find the area of the region that lies outside of the circle $x^2+y^2=9$ but inside of the circle $x^2+y^2-2y=48$ to two decimal places.

The first equation can be written $x^2+y^2=3^2$, the circle centered at the origin with radius 3.

Next lets find the standard form equation for the second circle:

$$x^2 + y^2 - 2y + \boxed{1} = 48 + \boxed{1}$$

$$x^2 + \left(y - \boxed{1}\right)^2 = 49$$

$$x^2 + (y-1)^2 = 7^2$$

So this is the circle centered at the point (0, 1) with radius 7.

Notice that the first smaller circle is entirely within the larger circle, so the area we want is simply:

$$\text{circle "area of big circle"} \; - \; \text{"area of small circle"} = \pi \cdot 7^2 - \pi \cdot 3^2 = 49\pi - 9\pi = \boxed{40\pi}$$

Rounding off to the hundredth place, we get:

$$40\pi \doteq 125.66370614359172954 \cong \boxed{125.66}$$

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27. A city has streets that run north and south, and avenues that run east and west, all equally spaced. Streets and avenues are numbered sequentially, as shown in the figure. Find the walking distance and the straight-line distance between the corner of 4th St. and 2nd Ave. and the corner of 9th St. and 14th Ave.

In other words, find the distance between the point (4, 2) and the point (9, 14):

$$\text{Distance} = \sqrt{(9-4)^2 + (14-2)^2} = \sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169} = \boxed{13}$$

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32. Determine the correct equation for the line passing through the point (1, 17) and the point (13, 4).

First we find the slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-17}{13-1} = \boxed{-\dfrac{13}{12}}$

So our line is of the form, $y = mx + b = -\dfrac{13}{12} \cdot x + b$, and we need to find b. But we know the line goes through the point (1, 17), which means when x=1 then we have y=17. Substituting into our last equation, we can solve for b:

$$y = -\frac{13}{12} \cdot x + b \quad \Rightarrow \quad 17 = -\frac{13}{12} \cdot 1 + b \quad \Rightarrow \quad 17 + \frac{13}{12} = b \quad \Rightarrow \quad \boxed{b = \frac{217}{12}}$$

Thus, in slope-intercept form our line's equation is $y = -\dfrac{13}{12} \cdot x + \dfrac{217}{12}$. To get this line into same form as the answer selections given, multiply both sides by 12, and get everything on the same side of the equal sign:

$$12y = -13x + 217 \quad \Rightarrow \quad \boxed{13x + 12y - 217 = 0}$$

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33. Determine the correct equation for the line with an *x*-intercept of –1 and *y*-intercept 8.

Our line goes through the points (-1, 0) and (0, 8). The slope is thus:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{0-(-1)} = \frac{8}{1} = \boxed{8}$$

We are told the y-intercept is 8. So, the slope-intercept equation for this line is:

$$y = mx + b \quad \Rightarrow \quad \boxed{y = 8x + 8}$$

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36. Determine the equation that expresses *A* is proportional to *G* and inversely proportional to *z*.
Symbols *a*, *b*, and *c* are constants.

$A = \dfrac{kG}{z}$ is an equation expressing this proportionality, with *k* as the constant of proportionality.

This may also be written as $A = k \cdot \dfrac{G}{z}$. If we replace the constant k with one of the constants given in the problem, it becomes: $\boxed{A = c \cdot \dfrac{G}{z}}$

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37. Express the statement as a formula: "*s* is inversely proportional to the square root of *t*".
Use the information that if *s* = 2 then *t* = 64 to find the constant of proportionality.

$$s = \frac{k}{\sqrt{t}} \quad \Rightarrow \quad 2 = \frac{k}{\sqrt{64}} \quad \Rightarrow \quad 2 = \frac{k}{8} \quad \Rightarrow \quad \boxed{16 = k}$$

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38. The resistance *R* of a wire varies directly as its length *L* and inversely as the square of its diameter *d*.
Find the constant of proportionality *K* if a wire 63.7 m long and 0.007 m in diameter has a resistance of 91 ohms.
Find the resistance R_1 of a wire made of the same material that is 1 m long and has a diameter of 0.002 m.

$$R = \frac{kL}{d^2} \quad \Rightarrow \quad 91 = \frac{k \cdot 63.7}{(0.007)^2} \quad \Rightarrow \quad k = \frac{91(0.007)^2}{63.7} \quad \Rightarrow \quad \boxed{k = 0.00007}$$

$$R = \frac{0.00007\,L}{d^2} \quad \Rightarrow \quad R_1 = \frac{(0.00007)\cdot 1}{(0.002)^2} \quad \Rightarrow \quad \boxed{R_1 = 17.5\,\Omega}$$

(Note: Ω is the symbol (a Greek letter) used to represent ohms.)

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39. In the short growing season of the Canadian arctic territory of Nunavut, some gardeners find it possible to grow gigantic cabbages in the midnight sun. Assume that the final size of a cabbage is proportional to the amount of nutrients it receives, and inversely proportional to the number of other cabbages surrounding it. A cabbage that received 20 oz of nutrients and had 6 other cabbages around it grew to 27 lb.

What size would it grow to if it received 10 oz of nutrients and had only 3 cabbage neighbors?

Let S be the final cabbage size, and let N be the amount of nutrients it receives, and let C be the number of neighboring cabbages. Then we are told that:

$$S = \frac{kN}{C} \quad \Rightarrow \quad 27 = \frac{k \cdot 20}{6} \quad \Rightarrow \quad k = \frac{27(6)}{20} = \frac{81}{10} \quad \Rightarrow \quad \boxed{k = 8.1}$$

$$S = \frac{8.1N}{C} \quad \Rightarrow \quad S = \frac{8.1(10)}{3} = \frac{81}{3} \quad \Rightarrow \quad \boxed{S = 27}$$

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40. The heat experienced by a hiker at a campfire is proportional to the amount of wood on the fire, and inversely proportional to the cube of his distance from the fire. If he is 23 ft from the fire, and someone doubles the amount of wood burning, approximately how far from the fire would he have to be so that he feels the same heat as before?

Let H_0 be the ***initial*** heat, and let W_0 be the ***initial*** amount of wood on the fire, and let d be the distance asked for in the problem. Let H, W, and D be any possible heat, wood, and distance…

We are told that $H = \frac{kW}{D^3}$. This is true of the original situation, $H_0 = \frac{kW_0}{23^3} \quad \Rightarrow \quad \boxed{H_0 = \frac{kW_0}{12167}}$,

and is also true of the new distance asked for in the problem: $H_0 = \frac{k(2W_0)}{d^3} \quad \Rightarrow \quad \boxed{H_0 = \frac{2kW_0}{d^3}}$

We used the same initial heat for both equations, since that's what the posed question calls for..

Since $H_0 = \frac{kW_0}{12167}$, and also $H_0 = \frac{2kW_0}{d^3}$, we must have that $\frac{kW_0}{12167} = \frac{2kW_0}{d^3}$. Cross-multiplying, we get:

$$kW_0 d^3 = 12167 \cdot 2kW_0 \quad \Rightarrow \quad kW_0 d^3 = 24334kW_0 \quad \Rightarrow \quad \frac{kW_0 d^3}{kW_0} = \frac{24334kW_0}{kW_0}$$

$$\Rightarrow \quad d^3 = 24334 \quad \Rightarrow \quad d = \sqrt[3]{24334} \doteq 28.97818414758208279 \cong \boxed{29} \text{ ft}$$
