

Precalculus Chapter 2 Quiz Part I (sections 1 – 4)

Most-Missed Problem Solutions (# 6-11, 13, 14, 16, 23-25, 27-31, 33, 34, 37, 39, 40)

6. For the function $f(x) = 3x^2 + 7$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$, $h \neq 0$.

$$\begin{aligned}\frac{f(a+h) - f(a)}{h} &= \frac{[3(a+h)^2 + 7] - [3a^2 + 7]}{h} = \frac{[3(a^2 + 2ah + h^2) + 7] - [3a^2 + 7]}{h} \\ &= \frac{3a^2 + 6ah + 3h^2 + 7 - 3a^2 - 7}{h} = \frac{\cancel{3a^2} + 6ah + 3h^2 + \cancel{7} - \cancel{3a^2} - \cancel{7}}{h} \\ &= \frac{6ah + 3h^2}{h} = \frac{h(6a + 3h)}{h} = \frac{\cancel{h}(6a + 3h)}{\cancel{h}} = \boxed{6a + 3h}\end{aligned}$$

7. For the function $f(x) = 4x^3$, find the difference quotient $\frac{f(a+h) - f(a)}{h}$.

$$\begin{aligned}\frac{f(a+h) - f(a)}{h} &= \frac{4(a+h)^3 - 4a^3}{h} = \frac{4(a^3 + 3a^2h + 3ah^2 + h^3) - 4a^3}{h} \\ &= \frac{4a^3 + 12a^2h + 12ah^2 + 4h^3 - 4a^3}{h} = \frac{\cancel{4a^3} + 12a^2h + 12ah^2 + 4h^3 - \cancel{4a^3}}{h} \\ &= \frac{12a^2h + 12ah^2 + 4h^3}{h} = \frac{h(12a^2 + 12ah + 4h^2)}{h} = \frac{\cancel{h}(12a^2 + 12ah + 4h^2)}{\cancel{h}} \\ &= \boxed{12a^2 + 12ah + 4h^2}\end{aligned}$$

8. Find the domain of the function $f(x) = 3x$, $-6 \leq x \leq 8$:

This function's domain is given. The domain is $[-6, 8]$. I also gave credit for $[-\infty, \infty]$, since the given function is usually defined over this interval, and I don't recall answering a question quite like this one in class.

9. What is the domain of the function $f(x) = \frac{x+6}{x^2-9}$?

The domain of f consists of all x such that $x^2 - 9 > 0 \Rightarrow (x+3)(x-3) > 0$.

Making our little +- chart:

$x + 3$	-	0	+	+
$x - 3$	-		-	0
Product	+	0	-	0
		-3	3	

The domain of f is $(-\infty, -3) \cup (3, \infty)$.

10. Find the domain of the function $f(x) = \sqrt[9]{x-4}$:

This is an odd radical, so can have negatives under the radical. For example, $\sqrt[9]{-1} = -\sqrt[9]{1} = -1$.

So the domain of this function is all real numbers, $(-\infty, \infty)$.

11. Find the domain of the function $f(x) = \frac{\sqrt{x}}{x^2 - 16x + 48}$.

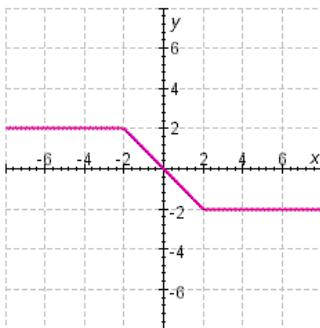
From the numerator, we must restrict x to non-negative values, so as to avoid leaving the realm of the real numbers. We must also exclude any values of x for which the denominator is 0:

$$x^2 - 16x + 48 = (x-12)(x-4) = 0 \Rightarrow x = 12 \text{ or } x = 4$$

So the domain of f consists of all non-negative real numbers, except for 4 and 12.

So the domain of this function is $(0, 4) \cup (4, 12) \cup (12, \infty)$.

13. Sketch the graph of the piecewise defined function: $f(x) = \begin{cases} 2 & \text{if } x < -2 \\ -x & \text{if } -2 \leq x \leq 2 \\ -2 & \text{if } x > 2 \end{cases}$



14. Determine whether the equation $x^2 + 3y = 3$ defines y as a function of x .

It defines a function, since you can solve for y as a function of x :

$$x^2 + 3y = 3 \Leftrightarrow 3y = 3 - x^2 \Leftrightarrow y = \frac{3 - x^2}{3}$$

16. Consider a family of functions $f(x) = 2(x - c)^3$.

Graph all the given members of the family in the viewing rectangle indicated.

(a) $c = 0, 2, 4, 6$; $[-10, 10]$ by $[-20, 20]$ (b) $c = 0, -2, -4, -6$; $[-10, 10]$ by $[-20, 20]$

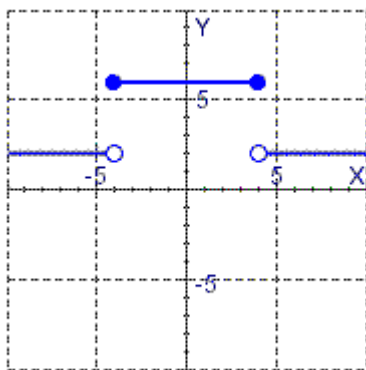
How do these values of c affect the graph?

For (a), subtracting positive values of c shifts the graph of $y = 2x^3$ to the right by c .

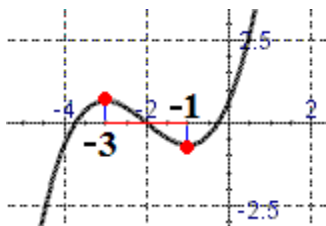
For (b), subtracting negative values of c from x is really adding something to x , so the graph of $y = 2x^3$ is shifted to the left by c .

23. Select the correct graph of the function $f(x) = \begin{cases} 6 & \text{if } |x| \leq 4 \\ 2 & \text{if } |x| > 4 \end{cases}$:

Since $|x| \leq 4 \Rightarrow -4 \leq x \leq 4$, the graph is thus:



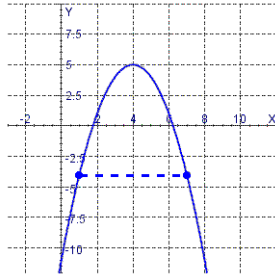
24. The function depicted,



Is decreasing over the interval $(-3, -1)$.

25. The graph of a function is given below.

What is the average rate of change of the function between the indicated values of the variable?



The average rate of change of y with respect to x is simply the change in y divided by the change in x , which is the slope. The two points shown in the graph have the same y -coordinate, so the change in y is 0, the slope is 0, i.e., the average rate of change is 0.

27. Determine the average rate of change for the function $f(x) = \frac{1}{x}$ between the $x = 1$ and $x = 6$.

$$= \frac{f(6) - f(1)}{6 - 1} = \frac{\frac{1}{6} - \frac{1}{1}}{5} = \frac{\left(\frac{-5}{6}\right)}{\left(\frac{5}{1}\right)} = -\frac{5}{6} \cdot \frac{1}{5} = \boxed{-\frac{1}{6}}$$

28. Find the average rate of change for the function $f(x) = \frac{1}{7}x + 3$ between $x = a$ and $x = a + h$.

The easy way to do this problem is to recognize that the average rate of change is the slope of the line, $\boxed{\frac{1}{7}}$.

Or, you may use the formula:

$$\begin{aligned} &= \frac{f(a+h) - f(a)}{a+h-a} = \frac{\left[\frac{1}{7}(a+h)+3\right] - \left[\frac{1}{7}a+3\right]}{h} = \frac{\left[\frac{1}{7}a + \frac{1}{7}h + 3\right] - \left[\frac{1}{7}a + 3\right]}{h} \\ &= \frac{\frac{1}{7}a + \frac{1}{7}h + 3 - \frac{1}{7}a - 3}{h} = \frac{\frac{1}{7} \cdot h}{h} = \boxed{\frac{1}{7}} \end{aligned}$$

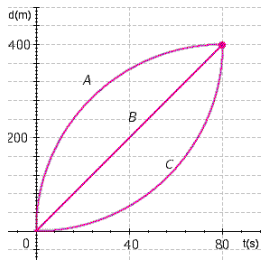
29. The table shows the number of CD players sold in a small electronics store in the years 1989 - 1999 as follows:

Year	CD players sold
1989	545
1999	695

What was the average rate of change of sales between 1989 and 1999?

$$= \frac{695 - 545}{1999 - 1989} = \frac{150}{10} = \boxed{15} \quad [\text{CDs / year}]$$

30. A 100-m race ends in a three-way tie for first place. The graph shows distance as a function of time for each of the three winners. Find the average speed for each winner.



Notice that the beginning point and ending points are the same for all three functions. They all begin at (0,0) and end at (80,400). So they all have the same average speed:

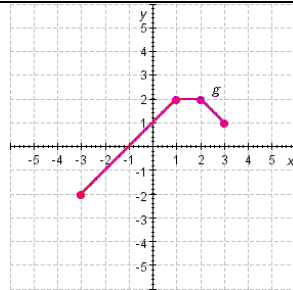
$$= \frac{400 - 0}{80 - 0} = \frac{400}{80} = \boxed{5} \quad [\text{m/s}]$$

31. Suppose the graph of f is given. Describe how the graph of $y = f(-x)$ can be obtained from the graph of f .

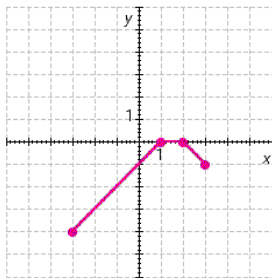
Reflect the graph of $y = f(x)$ about the y -axis.

33. The graph of g is given:

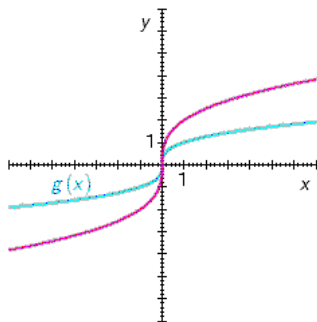
Find the graph of the function
 $y = g(x) - 2$



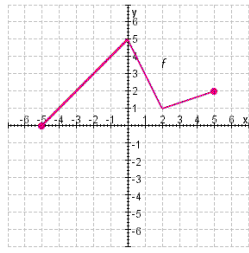
The desired graph is the one above, but shifted down by 2:



34. Sketch the graph of $g(x) = \sqrt[3]{x}$ by plotting points. Use the graph of g to sketch the graph $y = 2\sqrt[3]{x}$.



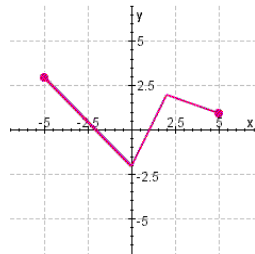
37. The graph of f is given below.



Find the graph of the function $y = -f(x) + 3$:

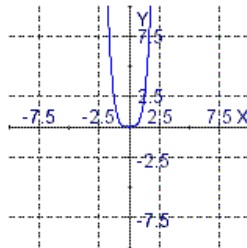
First, reflect the graph of f through the x -axis (that's the graph of $y = -f(x)$).

Then shift the graph up by 3 units, to obtain:



39. Sketch the function $g(x) = |x^4|$.

This is the same as the graph of $y = x^4$, since the absolute value of a non-negative number doesn't change the number:



40. A taxi company charges \$3.00 for the first mile (or part of a mile) and 30 cents for each succeeding two-tenths of a mile (or part). Express the cost C (in dollars) of a ride as a function of the distance x traveled (in miles) for $0 < x < 2$.

The correct answer wasn't given, but it should have been:

$$C(x) = \begin{cases} 3.00 & \text{for } 0 < x \leq 1 \\ 3.30 & \text{for } 1 < x \leq 1.2 \\ 3.60 & \text{for } 1.2 < x \leq 1.4 \\ 3.90 & \text{for } 1.4 < x \leq 1.6 \\ 4.20 & \text{for } 1.6 < x \leq 1.8 \\ 4.50 & \text{for } 1.8 < x < 2.0 \end{cases}$$