**Precalculus Chapter 3 Quiz – Solutions to the Most-Missed Problems,**
**# 8,12,13,21,23,27,28,32,34,39,43-47,49-51,53,54,56-60**

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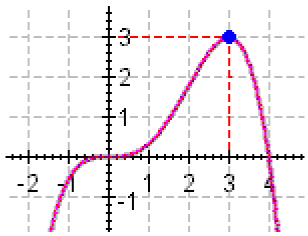
8. The graph of a polynomial function is given.




This polynomial has one local maximum which is also an absolute maximum (from the look of it) at the point (3,3).

There are no other relative (or local) maximums or minimums.

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12. What is the largest product *A* and *B* can have, given that $2A + 5B = 100$ ?

Solve the given equation for *B* in terms of *A*, and you get: $B = \frac{100-2A}{5}$ . Thus the product of *A* and *B* is:

$$AB = A\cdot\frac{100-2A}{5} = \frac{100A-2A^2}{5} = 20A - \frac{2}{5}A^2 = -\frac{2}{5}A^2 + 20A$$

Thus the product as a function of *A* is given by the quadratic polynomial:

$$P(A) = -\frac{2}{5}A^2 + 20A$$

The graph of this polynomial is a downward opening parabola, so its vertex will yield the maximum product.

"The *A*-coordinate of the vertex" = $A = \frac{-b}{2a} = \frac{-20}{2\left(-\frac{2}{5}\right)} = \frac{10}{\left(\frac{2}{5}\right)} = \frac{50}{2} = 25.$

That's the value of *A* that yields the the maximum product. Earlier we found that $B = \frac{100-2A}{5}$, hence

$$B = \frac{100-2\cdot 25}{5} = \frac{100-50}{5} = \frac{50}{5} = 10,$$

and the maximum product is

$$P_{\max} = A\cdot B = 25\cdot 10 = \boxed{250}$$

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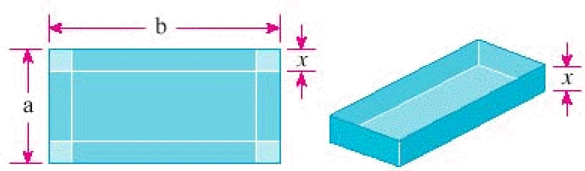
13. An open rectangular box is to be constructed from a piece of cardboard *a* = 14 cm by *b* = 43 cm, by cutting squares of length x from each corner and folding up the sides, as shown in the figure. What is the maximum volume to the nearest centimeter of such a box?




$$V = x(14-2x)(43-2x) = x\left(602-114x+4x^2\right) = 4x^3 - 114x^2 + 602x$$

Unfortunately, this one can't be done without the use of some technology (like a graphing calculator) or by using calculus. I've counted this one as extra credit. If you missed this one, it won't count against you; if you got it correct, I'll give you extra-credit.

21. Find a polynomial of degree 3 that has zeros of 2, –4, and 4, and where the coefficient of $x^2$ is 6.

$$P(x) = a(x-2)(x+4)(x-4) = a(x-2)(x^2-16) = a(x^3-2x^2-16x+32) = ax^3-2ax^2-16ax+32a$$

We are told that $-2a = 6 \quad \Rightarrow \quad \boxed{a = -3}$. Thus, $\boxed{P(x) = -3x^3+6x^2+48x-96}$.

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23. Use Descartes' Rule of Signs to determine how many positive and how many negative real zeros the polynomial can have. Then determine the possible total number of real zeros: $P(x) = 3x^3 - x^2 + 2x - 8$

This expression has 3 changes in sign, as we move from one coefficient to the next. This means the number of possible positive roots (zeros) is 3 ***or*** 3 – 2 = 1.

$$P(-x) = -3x^3 - x^2 - 2x - 8$$

This expression has 0 changes in sign, as we move from one coefficient to the next. So there are 0 possible negative roots. Thus,

$$\boxed{\text{1 or 3 positive roots}}$$

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27. Find all rational zeros of the polynomial $P(x) = x^4 - 29x^2 + 100$.

Let $u = x^2$, so $u^2 = x^4$. Then $P(x) = u^2 - 29u + 100 = 0$. Then

$$u = \frac{-(-29) \pm \sqrt{(-29)^2 - 4\cdot 1\cdot 100}}{2\cdot 1} = \frac{29 \pm \sqrt{841-400}}{2} = \frac{29 \pm \sqrt{441}}{2} = \frac{29 \pm 21}{2} = \boxed{25 \text{ or } 4}$$

Thus $u = x^2 = 25 \quad \Rightarrow \quad \boxed{x = \pm 5}$, and $u = x^2 = 4 \quad \Rightarrow \quad \boxed{x = \pm 2}$.

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28. Find all rational zeros of the polynomial $P(x) = x^4 + 11x^3 + 29x^2 - 11x - 30$.

a. $x = -1,\ x = 1,\ x = 5,\ x = -6$
b. $x = 1,\ x = 3,\ x = -5,\ x = -6$
c. $x = -1,\ x = -5,\ x = -6$
d. $x = -1,\ x = 1,\ x = -5,\ x = -6$
e. $x = -1,\ x = 1,\ x = -5,\ x = 6$

Any rational zeros must be factors of 30. All the given possible answers are, so this fact won't help us. We will have to do some synthetic division to eliminate possible answers…

$$\begin{array}{r|rrrrr} -1 & 1 & 11 & 29 & -11 & -30 \\ & & -1 & -10 & -19 & 30 \\ \hline & 1 & 10 & 19 & -30 & 0 \end{array}$$

The remainder is 0, so –1 is a rational zero of this polynomial, ruling out (b).

This means that $P(x) = (x+1)(x^3+10x^2+19x-30)$, and we can focus on the trinomial instead of the original quartic polynomial…

$$\begin{array}{r|rrrr} 1 & 1 & 10 & 19 & -30 \\ & & 1 & 11 & 30 \\ \hline & 1 & 11 & 30 & 0 \end{array}$$

The remainder is 0, so 1 is a rational zero of this polynomial, ruling out (c).

And now we know that $P(x)=(x+1)(x-1)(x^2+11x+30)$. But that last quadratic factor factors into linear factors:

$$P(x)=(x+1)(x-1)(x+5)(x+6)$$

So the rational roots are $\boxed{x=-1, \quad x=1, \quad x=-5, \quad x=-6}$.

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32. Find all the real zeros of the polynomial $P(x) = 5x^4 + 36x^3 + 47x^2 - 52x - 12$.

a. $x = -3,\ x = -\frac{1}{5},\ x = 2 \pm 2\sqrt{2}$

b. $x = 3,\ x = -\frac{1}{5},\ x = -2 \pm 2\sqrt{2}$

c. $x = -3,\ x = -\frac{1}{5},\ x = -2 \pm 2\sqrt{2}$

d. $x = 3,\ x = \frac{1}{5},\ x = -2 - 2\sqrt{2}$

e. $x = -3,\ x = \frac{1}{5},\ x = -2 \pm 2\sqrt{2}$

This problem is a process of elimination. First we can rule out (d), since if $-2-2\sqrt{2}$ were a zero, then $-2+2\sqrt{2}$ would have to be also, or there would be some radicals appearing in the polynomial's coefficients.

So let's commence synthetic division to eliminate more candidates…

$$\begin{array}{r|rrrrr} -3 & 5 & 36 & 47 & -52 & -12 \\ & & -15 & -63 & 48 & 12 \\ \hline & 5 & 21 & -16 & -4 & 0 \end{array}$$

The remainder is 0, so –3 is a rational zero of this polynomial, ruling out (b).

This means that $P(x)=(x+3)(5x^3+21x^2-16x-4)$, and we can focus on the trinomial instead of the original

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & 21 & -16 & -4 \\ & & -1 & -4 & 4 \\ \hline & 5 & 20 & -20 & 0 \end{array}$$

The remainder is 0, so $-\frac{1}{5}$ is a rational zero of this polynomial, ruling out (e).

Thus, $P(x)=(x+3)\left(x+\frac{1}{5}\right)(5x^2+20x-20)=(x+3)\left(x+\frac{1}{5}\right)5(x^2+4x-4)=(x+3)(5x+1)(x^2+4x-4)$ .

That last quadratic factor's roots can be found using the quadratic formula:

$$x=\frac{-4\pm\sqrt{16-4\cdot 1\cdot(-4)}}{2\cdot 1}=\frac{-4\pm\sqrt{16+16}}{2}=\frac{-4\pm\sqrt{16(1+1)}}{2}=\frac{-4\pm 4\sqrt{2}}{2}=-2\pm 2\sqrt{2}$$

So the real roots are $\boxed{x=-3,\quad x=-\frac{1}{5},\quad x=-2+2\sqrt{2},\quad x=-2-2\sqrt{2}}$.

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34. Find integers that are upper and lower bounds for the real zeros of $P(x)=x^3-18x^2+67x+30$.

a. $x=-18,\ x=-1$
b. $x=1,\ x=18$
c. $x=-1,\ x=18$
d. $x=-1,\ x=-18$
e. $x=0,\ x=1$

Two possible positive upper bounds are given, 1 and 18. So we'll try these first (using synthetic division):

| | | | | |
|---|---|---|---|---|
| 1 | 1 | −18 | 67 | 30 |
| | | 1 | −17 | 50 |
| | 1 | −17 | 50 | 80 |

That last row has a negative entry, so 1 isn't an upper bound on the roots.

So next we'll check on the given possible negative root bound (it's now between (b) & (c)), –1.

| | | | | |
|---|---|---|---|---|
| −1 | 1 | −18 | 67 | 30 |
| | | −1 | 19 | −86 |
| | 1 | −19 | 86 | −56 |

That last row has alternating +-+- entries, so –1 is a lower bound on the roots.

We conclude that the real roots of $P(x)$ are between $\boxed{-1 \text{ and } 18}$.

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39. Evaluate the expression $i^{64}$ and write the result in the form *a* + *bi*.

The key is to realize that since $i^2=-1$, then $i^4=i^2\cdot i^2=(-1)(-1)=1$. But $64=4\cdot 16$, so

$$i^{64}=\left(i^4\right)^{16}=1^{16}=\boxed{1}.$$

In general, to find $i^n$, divide *n* by 4, and note the remainder, *r*. Then $i^n=i^r$.
In the present case, the remainder is 0 (upon dividing 64 by 4), so $i^{64}=i^0=1$.

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43. Find all solutions of the equation $z+8+\frac{20}{z}=0$ and express them in the form *a* + *bi*. In general

Multiply both sides of the equation by *z* to cancel out that denominator: $z^2+8z+20=0$

Invoking the quadratic formula: $z=\frac{-8\pm\sqrt{64-4\cdot 1\cdot 20}}{2\cdot 1}=\frac{-8\pm\sqrt{64-80}}{2}=\frac{-8\pm\sqrt{-16}}{2}=\frac{-8\pm i4}{2}=\boxed{-4\pm 2i}$

44. $P(x)=x^4-5x^2-6$. Factor $P$ completely.

Let $u=x^2$, so $u^2=x^4$. Then $P(x)=u^2-5u-6=(u+1)(u-6)=0$.

Thus $u=x^2=-1 \Rightarrow \boxed{x=\pm i}$, and $u=x^2=6 \Rightarrow \boxed{x=\pm\sqrt{6}}$. And so,

$$\boxed{P(x)=\left(x+\sqrt{6}\right)\left(x-\sqrt{6}\right)(x+i)(x-i)}$$

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45. Factor the $P(x)=x^4+14x^2+49$ completely and find all its zeros. State the multiplicity of each zero.

Let $u=x^2$, so $u^2=x^4$. Then

$$P(x)=u^2+14u+49=(u+7)^2=\left(x^2+7\right)^2=\left(x^2-\left(i\sqrt{7}\right)^2\right)^2$$
$$=\left(\left(x+i\sqrt{7}\right)\left(x-i\sqrt{7}\right)\right)^2=\left(x+i\sqrt{7}\right)^2\left(x-i\sqrt{7}\right)^2$$

So $P$ has two complex roots at $\boxed{i\sqrt{7},-i\sqrt{7}\quad \text{(each with multiplicity 2)}}$.

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46. Factor the polynomial $P(x)=x^5+14x^3+49x$ completely and find all its zeros.

$$P(x)=x^5+14x^3+49x=x\left(x^4+14x^2+49\right)$$

Notice that in problem #44 we found the zeros of that trinomial factor. All that's new is the factor of *x*, which results in a new root, 0.

So $P$ has zeros (roots) at $\boxed{i\sqrt{7},-i\sqrt{7},\ 0}$.

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47. Find the polynomial $P(x)$ of degree 4 with integer coefficients, and zeros $3-3i$ and 2 with 2, a zero of multiplicity 2.

$$P(x)=(x-(3-3i))(x-(3+3i))(x-2)^2$$

We got the "blue" factor for free, since roots of polynomials with real coefficients always come in complex conjugate pairs. Thus

$$P(x)=(x-(3-3i))(x-(3+3i))(x-2)^2=\left(x^2-(3+3i)x-(3-3i)x+(3-3i)(3+3i)\right)(x-2)^2$$
$$=\left(x^2-6x+18\right)\left(x^2-4x+4\right)=\boxed{x^4-10x^3+46x^2-96x+72}$$

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49. Factor the polynomial $P(x)=x^4-16x^2-225$ completely into linear factors with complex coefficients.

Let $u=x^2$, so $u^2=x^4$. Then $P(x)=u^2-16u-225=(u+9)(u-25)=\left(x^2+9\right)\left(x^2-25\right)$, and we can write each of these binomials as a difference of squares:

$$P(x)=(x^2+3^2)(x^2-5^2)=\left(x^2-(3i)^2\right)(x^2-5^2)=\boxed{(x+3i)(x-3i)(x+5)(x-5)}$$

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50. Find the *x*- and *y*-intercepts of the rational function: $y=\dfrac{x-6}{x+6}$

Any *x*-intercepts occur when *y*=0: $0=\dfrac{x-6}{x+6} \Rightarrow 0=x-6 \Rightarrow x=6 \Rightarrow \boxed{(6,0)}$

Any *y*-intercepts occur when *x*=0: $y=\dfrac{0-6}{0+6}=-1 \Rightarrow \boxed{(0,-1)}$

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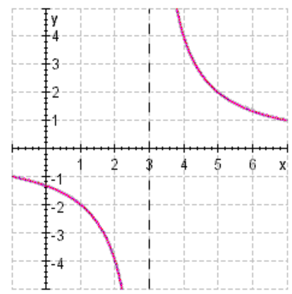
51. Use transformations of the graph of $y=\dfrac{1}{x}$ to graph the rational function $r(x)=\dfrac{1}{x-3}$

The graph of $y=\dfrac{1}{x}$ is shifted to the right by 3 to get the graph of $r(x)=\dfrac{1}{x-3}$:

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53. Find the intercepts and asymptotes of the rational function: $y=\dfrac{6x+24}{-4x+4}=\dfrac{2(3x+12)}{2(-2x+2)}=\dfrac{3x+12}{-2x+2}$

Any *x*-intercepts occur when *y*=0: $0=\dfrac{3x+12}{-2x+2} \Rightarrow 0=3x+12 \Rightarrow x=-4 \Rightarrow \boxed{(-4,0)}$

Any *y*-intercepts occur when *x*=0: $y=\dfrac{3\cdot 0+12}{-2\cdot 0+2}=6 \Rightarrow \boxed{(0,6)}$

To find the vertical asymptote: $-2x+2=0 \Rightarrow \boxed{x=1}$ .

To find the horizontal asymptote, we divide: $y=\dfrac{3x+12}{-2x+2}=\dfrac{3}{-2}\cdot\dfrac{x+4}{x+1}=\left(-\dfrac{3}{2}\right)\cdot\dfrac{x+4}{x+1}$ . Using synthetic division:

$$\begin{array}{r|rr} -1 & 1 & 4 \\ & & -1 \\ \hline & 1 & 3 \end{array}$$

Thus, $y=\left(-\dfrac{3}{2}\right)\cdot\dfrac{x+4}{x+1}=\left(-\dfrac{3}{2}\right)\cdot\left(1+\dfrac{3}{x+1}\right)=-\dfrac{3}{2}-\dfrac{9}{2(x+1)}$.

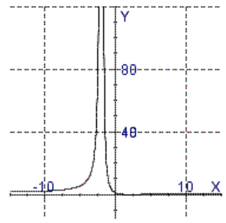
As $x$ gets large in a positive or negative direction, the denominator of the second term gets large, causing that term to shrink and get closer and closer to 0. That leaves the horizontal asymptote: $\boxed{y=-\dfrac{3}{2}}$.

54. Determine the correct graph of the rational function $y = \dfrac{x^2 - 4x + 4}{x^2 + 4x + 4} = \dfrac{(x-2)^2}{(x+2)^2}$

This function has one vertical asymptote at $x = -2$, and an $x$-intercept at $x = 2$.

Also this function is never negative, due to all the squares.

Thus:

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56. Given the function, $y = \dfrac{x^2 + 7x + 11}{x - 5}$, identify another function with the same end behavior.

Using synthetic division:

| | | | |
|---|---|---|---|
| 5 | 1 | 7 | 11 |
| | | 5 | 60 |
| | 1 | 12 | 71 |

Thus, $y = \dfrac{x^2 + 7x + 11}{x - 5} = x + 12 + \dfrac{71}{x - 5}$, and we have the slant asymptote: $\boxed{y = x + 12}$.

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57. Find a polynomial that has the same end behavior as the rational function: $y = \dfrac{7x^5}{x^3 - 9}$

We must use polynomial long division on this one:

$$\begin{array}{r|l} & 7x^2 \\ \hline x^3 - 9 \;) & 7x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 0 \\ & -(\underline{7x^5 \qquad\qquad -63x^2}) \\ & \qquad\qquad\qquad 63x^2 + 0x + 0 \end{array}$$

Thus, $y = \dfrac{7x^5}{x^3 - 9} = 7x^2 + \dfrac{63x}{x^3 - 9}$.

We have the asymptotic quadratic: $\boxed{y = 7x^2}$.

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58. Find a polynomial that has the same end behavior as the rational function $y = \dfrac{x^4 - 9x^3 + 6}{x - 9}$.

Using synthetic division:

| | | | | | |
|---|---|---|---|---|---|
| 9 | 1 | −9 | 0 | 0 | 6 |
| | | 9 | 0 | 0 | 0 |
| | 1 | 0 | 0 | 0 | 6 |

Thus, $y = \dfrac{x^4 - 9x^3 + 6}{x - 9} = x^3 + \dfrac{6}{x - 9}$, and we have the asymptotic cubic: $\boxed{y = x^3}$.

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59. The rabbit population on Mr. Jenkins' farm follows the formula: $P(t)=\frac{5000t}{t+10}$. For this formula, t > 0 is the time in months since the beginning of the year. What is the eventual population of rabbits?

Using synthetic division:

$$\begin{array}{r|rr} -10 & 5000 & 0 \\ & & -50000 \\ \hline & 5000 & -50000 \end{array}$$

Thus, $P(t)=\frac{5000t}{t+10}=5000-\frac{50000}{t+10}$, and that last term approaches 0 as t gets large, approaching an eventual population of $\boxed{5000}$ rabbits.

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60. After a certain drug is injected into a patient, the concentration *C* of the drug in the bloodstream is monitored. At time t > 0 (in minutes since the injection), the concentration (in mg/L) is given by the equation:

$c(t)=\frac{20t}{t^2+2}$. What is the eventual concentration of the drug?

The denominator has a larger degree than the numerator, so this ration approaches 0 as t gets large, so the concentration approaches $\boxed{0}$.

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