Chapter 4 Quiz – Solutions to Most-Missed Problems

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14. If \$1,000 is invested at an interest rate of 10% per year, compounded monthly, find the amount of the investment at the end of 4 years.

$$A(t) = A_0\left(1+\frac{r}{N}\right)^{Nt} \quad \rightarrow \quad A(t) = 1000\left(1+\frac{0.10}{12}\right)^{12t}$$

$$\Rightarrow \quad A(4) = 1000\left(1+\frac{0.10}{12}\right)^{12\cdot 4} = 1000\left(1.08\bar{3}\right)^{48} = 1000\left(1.08\bar{3}\right)^{48} \cong 1489.354075 \cong \boxed{\$1,489.35}$$

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23. The age of an ancient artifact can be determined by the amount of radioactive carbon-14 remaining in it. If D_0 is the original amount of carbon-14 and D is the amount remaining, then the artifact's age A (in years) is given by $A = -8267\ln\left(\dfrac{D}{D_0}\right)$. Find the age of an object if the amount D of carbon-14 that remains in the object is 74% of the original amount D_0.

$$A = -8267\ln\left(\frac{D}{D_0}\right) \quad \rightarrow \quad A = -8267\ln(0.74) \cong -8267(-0.301105092783922161425)$$

$$A \cong 2489.235802044679985 \cong \boxed{2500 \text{ years old}}$$

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24. The rate at which a battery charges is slower the closer the battery is to its maximum charge C_0.

The time (in hours) required to charge a fully discharged battery to a charge C, is given by $t = -k\ln\left(1-\dfrac{C}{C_0}\right)$,

where k is a positive constant that depends on the battery. For a certain battery, k=0.23. If this battery is fully discharged, how long will it take to charge that is 81% of its maximum charge C_0? Round to four decimal places.

$$t = -k\ln\left(1-\frac{C}{C_0}\right) \quad \rightarrow \quad t = -0.23\ln(1-0.81) \cong -0.23(-1.660731206821650908)$$

$$t \cong 0.3819681775689797 \cong \boxed{0.3820 \text{ hours}}$$

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33. Solve the logarithmic equation for *x*. $\log_2 2 + \log_2 x = \log_2 3 + \log_2(x-5)$

$$\log_2 2 + \log_2 x = \log_2 3 + \log_2(x-5) \quad \Rightarrow \quad \log_2(2x) = \log_2(3(x-5))$$

$$\Rightarrow \quad 2x = 3(x-5) \quad \Rightarrow \quad \boxed{x = 15}$$

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34. Solve the logarithmic equation for *x*. $\log_3(x+4) - \log_3(x-4) = 3$

$$\log_3(x+4)-\log_3(x-4)=3 \;\Rightarrow\; \log_3\left(\frac{x+4}{x-4}\right)=3 \;\Rightarrow\; \frac{x+4}{x-4}=3^3=27$$

$$\Rightarrow\; x+4=27(x-4) \;\Rightarrow\; x+4=27x-108 \;\Rightarrow\; 112=26x \;\Rightarrow\; \boxed{x=\frac{56}{13}\approx 4.31}$$

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35. For what value of *x* is the following true? log (*x* + 9) = log *x* + log 9

$$\log(x+9)=\log x+\log 9 \;\Rightarrow\; \log(x+9)=\log(9x) \;\Rightarrow\; x+9=9x \;\Rightarrow\; \boxed{x=\frac{9}{8}=1.125}$$

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36. Solve the inequality. log (*x* – 2) + log (9 – *x*) < 1

Solved in class…

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38. A man invests $5,000 in an account that pays 8% interest per year, compounded quarterly.
Find the amount after 5 years.

$$A(t)=A_0\left(1+\frac{r}{N}\right)^{Nt} \;\rightarrow\; A(t)=5000\left(1+\frac{0.08}{4}\right)^{4t}$$

$$\Rightarrow\; A(5)=5000\left(1+\frac{0.08}{4}\right)^{4\cdot 5}=5000(1.02)^{28}\cong 7429.73697989\cong \boxed{\$7429.74}$$

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39. A sum of $3,000 was invested for 4 years, and the interest was compounded semiannually.
If this sum amounted to $5,000 in the given time, what was the interest rate?

$$A(t)=A_0\left(1+\frac{r}{N}\right)^{Nt} \;\rightarrow\; A(t)=3000\left(1+\frac{r}{2}\right)^{2t} \;\Rightarrow\; A(4)=3000\left(1+\frac{r}{2}\right)^{2\cdot 4}=5000$$

$$\Rightarrow\; \left(1+\frac{r}{2}\right)^{8}=\frac{5}{3} \;\Rightarrow\; 1+\frac{r}{2}=\sqrt[8]{\frac{5}{3}} \;\Rightarrow\; r=2\left(\sqrt[8]{\frac{5}{3}}-1\right)\cong 0.1318718221\cong \boxed{13.19\%}$$

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40. A small lake is stocked with a certain species of fish. The fish population is modeled by the function
$P=\dfrac{12}{1+4e^{-0.8t}}$, where *P* is the number of fish ***in thousands*** and *t* is measured in years since the lake was stocked.
After how many years will the fish population reach 5,000 fish?

$$P=\frac{12}{1+4e^{-0.8t}}=5 \;\Rightarrow\; 12=5\left(1+4e^{-0.8t}\right) \;\Rightarrow\; \frac{12}{5}=1+4e^{-0.8t} \;\Rightarrow\; \frac{12}{5}-1=4e^{-0.8t}$$

$$\Rightarrow\; \frac{7}{5}=4e^{-0.8t} \;\Rightarrow\; \frac{7}{20}=e^{-0.8t} \;\Rightarrow\; \ln\left(\frac{7}{20}\right)=-0.8t \;\Rightarrow\; t=\frac{\ln(0.35)}{-0.8}\cong \boxed{1.31\text{ years}}$$

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41. Suppose you're driving your car on a cold winter day (17°F outside) and the engine overheats (at about 218°F). When you park, the engine begins to cool down. The temperature T of the engine t minutes after you park satisfies the equation $\ln\left(\frac{T-17}{201}\right)=-0.11t$. Find the temperature of the engine after 20 min (t = 20).

$$\ln\left(\frac{T-17}{201}\right)=-0.11t \quad\rightarrow\quad \ln\left(\frac{T-17}{201}\right)=-0.11(20)=-2.2 \quad\Rightarrow\quad \frac{T-17}{201}=e^{-2.2}$$

$$\Rightarrow\quad T-17=201e^{-2.2} \quad\Rightarrow\quad T=17+201e^{-2.2}\cong \boxed{39°F}$$

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42. The population of a certain city was 118,000 in 1994, and the observed relative growth rate is 3% per year. In what year will the population reach 219,000?

The relative growth rate of a quantity is the growth rate of that quantity ***divided by*** that quantity. It's usually called *the relative rate of change*. For the exponential function $A(t)=A_0e^{rt}$, the relative rate of change turns out to be r (it takes some calculus to prove this, but we'll just accept this in this class). So in this case, r =3% = 0.03 .

$$A(t)=A_0e^{rt} \quad\rightarrow\quad A(t)=118e^{0.03t}=219 \quad\Rightarrow\quad e^{0.03t}=\frac{219}{118} \quad\Rightarrow\quad 0.03t=\ln\frac{219}{118}$$

$$\Rightarrow\quad t=\frac{\ln\frac{219}{118}}{0.03}\cong 20.6129035116945\cong 20.6 \quad\Rightarrow\quad 1994+20.6=2014.6 \quad\Rightarrow\quad \boxed{2014}$$

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44. An infectious strain of bacteria increases in number at a relative growth rate of 200% per hour. When a certain critical number of bacteria are present in the bloodstream, a person becomes ill. If a single bacterium infects a person, the critical level is reached in 33 hours, how long (in hours) will it take for the critical level to be reached if the same person is infected with 16 bacteria?

The relative growth rate of a quantity is the growth rate of that quantity **divided by** that quantity. It's usually called *the relative rate of change*. For the exponential function $A(t)=A_0e^{rt}$, the relative rate of change turns out to be r (it takes some calculus to prove this, but we'll just accept this in this class). So in this case, r =200% = 2.

$$A(t)=A_0e^{rt} \quad\rightarrow\quad A(t)=e^{2t} \quad\Rightarrow\quad A(33)=e^{2\cdot 33}\cong 46{,}071{,}866{,}343{,}312{,}915{,}426{,}773{,}184{,}428$$

$$A(t)=16e^{2t}\cong 46{,}071{,}866{,}343{,}312{,}915{,}426{,}773{,}184{,}428 \quad\Rightarrow\quad e^{2t}\cong\frac{46{,}071{,}866{,}343{,}312{,}915{,}426{,}773{,}184{,}428}{16}$$

$$\Rightarrow\quad 2t\cong\ln\left(\frac{46{,}071{,}866{,}343{,}312{,}915{,}426{,}773{,}184{,}428}{16}\right)$$

$$\cong\ln\left(2{,}879{,}491{,}646{,}457{,}057{,}214{,}173{,}324{,}026.75375543\right)$$

$$\Rightarrow\quad t\cong\frac{\ln\left(2{,}879{,}491{,}646{,}457{,}057{,}214{,}173{,}324{,}026.75375543\right)}{2}\cong\boxed{31.61\text{ hours}}$$

By the way, this problem is absurd! The number of bacteria that this problem says will make one ill is way too many. In fact, a typical bacteria weighs 10^{-12} grams. The number of bacteria that would make one ill would then weigh over 46 billion metric tons! That many bacteria would crush you under their immense weight!

45. The half-life of cesium-137 is 30 years. Suppose we have a 17-g sample.
Find a function that models the mass remaining after *t* years.

$$A(t) = A_0 e^{rt} \quad \rightarrow \quad A(t) = 17e^{rt} \quad \Rightarrow \quad A(30) = 17e^{r\cdot 30} = \frac{17}{2} \quad \Rightarrow \quad e^{30r} = \frac{1}{2}$$

$$\Rightarrow \quad 30r = \ln 0.5 \quad \Rightarrow \quad r = \frac{\ln 0.5}{30} \cong -0.0231 \quad \Rightarrow \quad \boxed{A(t) \cong 17e^{-0.023t}}$$

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46. Radium-221 has a half-life of 30 seconds. How long will it take for 95% of a sample to decay?

$$A(t) = A_0 e^{rt} \quad \rightarrow \quad A(t) = A_0 e^{rt} \quad \Rightarrow \quad A(30) = A_0 e^{r\cdot 30} = \frac{A_0}{2} \quad \Rightarrow \quad e^{30r} = \frac{1}{2}$$

$$\Rightarrow \quad 30r = \ln 0.5 \quad \Rightarrow \quad \boxed{r = \frac{\ln 0.5}{30}} \quad \Rightarrow \quad A(t) = A_0 e^{\left(\frac{\ln 0.5}{30}\right)t} = 0.05A_0$$

$$\Rightarrow \quad e^{\left(\frac{\ln 0.5}{30}\right)t} = 0.05 \quad \Rightarrow \quad \left(\frac{\ln 0.5}{30}\right)t = \ln 0.05 \quad \Rightarrow \quad t = \frac{\ln 0.05}{\left(\frac{\ln 0.5}{30}\right)} \cong \boxed{129.657 \text{ sec}}$$

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49. If one earthquake is 16 times as intense as another, how much larger is its magnitude on the Richter scale?

$$M = \log\left(\frac{16I}{S}\right) = \log\left(16\cdot\frac{I}{S}\right) = \log 16 + \log\left(\frac{I}{S}\right) = \log 16 + \log\left(\frac{I}{S}\right)$$

$$M \cong 1.20411998265592478 + \log\left(\frac{I}{S}\right) \cong \boxed{1.20} + (\text{magnitude of weaker quake})$$

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50. The noise from a power mower was measured at 102 dB. The noise level at a rock concert was measured at 122 dB. Find the ratio of the intensity of the rock music to that of the power mower.

$$B = 10\cdot\log\left(\frac{I_{mower}}{I_0}\right) = 102 \quad \Rightarrow \quad \log\left(\frac{I_{mower}}{I_0}\right) = 10.2 \quad \Rightarrow \quad \frac{I_{mower}}{I_0} = 10^{10.2} \quad \Rightarrow \quad \boxed{I_{mower} = 10^{10.2} I_0}$$

$$B = 10\cdot\log\left(\frac{I_{concert}}{I_0}\right) = 122 \quad \Rightarrow \quad \log\left(\frac{I_{concert}}{I_0}\right) = 12.2 \quad \Rightarrow \quad \frac{I_{concert}}{I_0} = 10^{12.2} \quad \Rightarrow \quad \boxed{I_{concert} = 10^{12.2} I_0}$$

Thus, $\frac{I_{concert}}{I_{mower}} = \frac{10^{12.2} I_0}{10^{10.2} I_0} = 10^{12.2-10.2} = 10^2 = \boxed{100}$
