Chapter 2 Section 1 Lesson
Kinds of Numbers

Introduction

This lesson briefly reviews the real numbers and introduces variables.

Digits

Digits are the ten number symbols used to write any number. These are the digits:

0 1 2 3 4 5 6 7 8 9

Counting Numbers

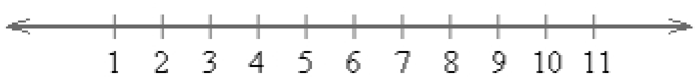
Counting numbers are the numbers we use when we count, and the rules of place value allow us to go beyond nine. They are also called "natural numbers." The counting numbers are

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 \ldots$$

Note:

- The . . . after the number 12 in the list above indicates that the counting numbers continue beyond 12, even though we've stopped listing them.

On the number line, the counting numbers can be represented as shown below.



This image represents an animation that can only be seen in the course online .

Note:

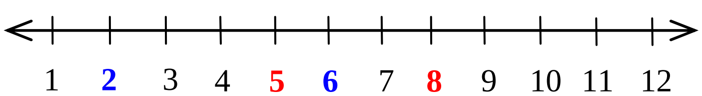
- Counting numbers do not include zero. The number line is usually drawn with arrows on both ends, but the counting numbers begin at 1 and do not include zero or anything to the left of 1.

Comparing Counting Numbers

Any number to the right of another number on the number line is said to be **greater than** the other number.

Any number to the left of another number on the number line is said to be **less than** the other number.

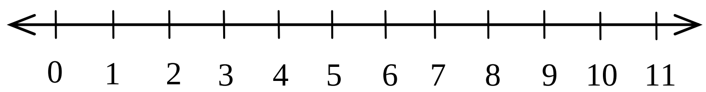
Examples: We say "6 is greater than 2" and we write $6 > 2$.
We can write "5 is less than 8" as $5 < 8$.



View the animation in the course online to see an additional example of this concept.

Whole Numbers

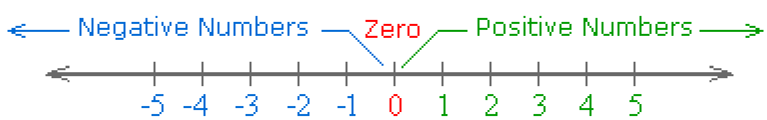
Whole numbers include the counting numbers, but they begin with zero.



The concepts “greater than” and “less than” also apply to whole numbers.

Integers

Whole numbers together with negative numbers are called **integers**. On a number line, negative numbers are to the left of zero, and positive numbers are to the right. Numbers that are the same distance from zero but in opposite directions are called **opposite numbers**. For example, the numbers 3 and -3 are opposites.



This image represents an animation that can only be seen in the course online .

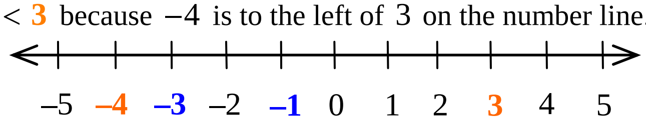
Note:

- The negative sign is always used with the negative numbers.
- No sign is used with positive numbers.
- Zero is neither positive nor negative.

Comparing Integers

The concepts greater than and less than also apply to integers, but be careful with the sign!

Examples: **–1** > **–3** because -1 is to the right of -3 on the number line.
–4 < **3** because -4 is to the left of 3 on the number line.



View the animation in the course online to see additional examples.

Rational Numbers

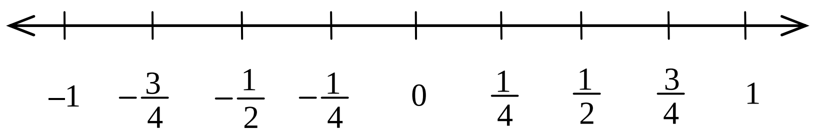
The set of **rational numbers** includes any number that can be written as a **fraction**.

Examples:

The number -3 is a rational number because it can be written as $\frac{-3}{1}$, which is a fraction. So, the set of rational numbers includes all integers.

The number $\frac{2}{7}$ is another rational number.

The number line below shows a few rational numbers.



The concepts greater than and less than also apply to fractions.

Examples: $\frac{1}{2} < \frac{3}{4}$

$-\frac{1}{2} > -\frac{3}{4}$

Improper Fractions

A fraction is called **improper** if the integer in the numerator is greater than the integer in the denominator. Students are often taught to convert improper fractions into **mixed number** notation, which consists of a whole number with a fraction to its right.

Example: The improper fraction $\frac{8}{3}$ is equal to the mixed number $2\frac{2}{3}$.

Mixed Numbers

In algebra we rarely use mixed number notation. Improper fractions are preferred. The algebraic expression ab means $a \times b$, but the mixed number $2\frac{2}{3}$ actually means $2 + \frac{2}{3}$. We avoid this notational discrepancy by simply avoiding mixed numbers. When confronted by a mixed number in an algebra class, the first step is almost always to convert it to an improper fraction.

For example, we can convert the mixed number $7\frac{3}{5}$ to an improper fraction using the following steps.

$$7\frac{3}{5} = 7 + \frac{3}{5} = \frac{7}{1}\left(\frac{5}{5}\right) + \frac{3}{5} = \frac{7 \times 5 + 3}{5} = \frac{35 + 3}{5} = \frac{38}{5}$$

Fractions in Decimal Form

Fractions may be written in decimal form. To convert a fraction into decimal form, we divide the numerator by the denominator using long division or a calculator.

Often we round the decimal that results. Note that rounding a decimal number results in an approximation to the fraction, rather than the true, exact value of the fraction.

When decimals have a repeating set of digits, we can indicate the repetition by either "$\ldots$" or by a bar over the repeating digits. (Note that the three dots are also used to indicate an infinite set of numbers that goes on without ending, even if there is no repetition.)

Examples:

$\frac{1}{2} = 0.5$	The decimal form is the exact value of the fraction.
$\frac{1}{3} \cong 0.33$	The decimal form is only an approximation of the fraction's exact value.
$\frac{1}{3} = 0.333\ldots$	The decimal form is the exact value of the fraction. The "$\ldots$" indicates endlessly repeating 3's.
$\frac{1}{3} = 0.\overline{3}$	This decimal notation is neater than the version above The bar indicates that the 3s endlessly repeat.

Irrational Numbers

Rational numbers can be written as terminating or repeating decimals. **Irrational numbers** have nonterminating, nonrepeating decimal expansions.

Together, rational and irrational numbers make up the set of **real numbers**. A straight line is a model of the set of real numbers; each point on the line corresponds to a unique real number.

Examples of real numbers follow.

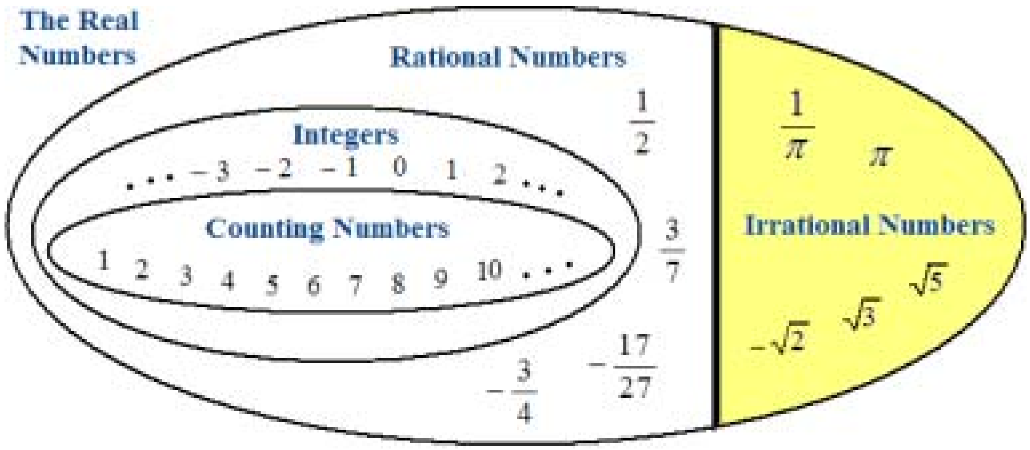
$\pi = 3.1415926535\ldots$	An irrational number
5 or $\frac{5}{1}$	A rational number
$\frac{3}{4}$ or 0.75	A rational number
$\sqrt{2} = 1.414213562\ldots$	An irrational number
$\frac{5}{7}$ or $0.\overline{714285}$	A rational number

Note:

- In the examples $\pi = 3.1415926535\ldots$ and $\sqrt{2} = 1.414213562\ldots$, the dots indicate that the decimal expansions continue forever, although in these cases not by the repetition of any simple patterns.

Real Numbers

The diagram below depicts the kinds of numbers that make up the set of real numbers, along with representatives of each type of number.



This diagram shows that all counting numbers are integers, and all integers are rational numbers, but that no irrational numbers are rational. All of these numbers are real numbers, which consist of the rational and irrational numbers combined. The animation in the course online may also help you visualize the set of real numbers.

Using Variables to Represent Numbers

In algebra, we use letters and other symbols to represent numbers. Letters that represent numbers in algebra are called **variables.**

The rules that govern how numbers behave also govern how variables behave in algebra. In fact, we'll see that the easiest way to describe the various properties of numbers is by using variables, the "common nouns" of algebra.

Note:

- You cannot take for granted what kind of number a variable may represent.

  Example: If the variable y represents any real number, that number could be positive, negative, zero, a whole number, a fraction, or an irrational number.

  Example: If the variable x represents a whole number, we know that x cannot be a fraction, and it cannot be negative.

End of Lesson