

## Chapter 2 Section 2 Lesson Behavior of Numbers and Variables

### Introduction

The rules that we use to perform operations on numbers also apply to operations involving variables. In this lesson, we review these rules. These rules, or “properties,” of the real numbers can be thought of as the rules of the game of algebra.

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### Commutative Property of Addition

Numbers or variables can be added in any order without affecting their sum. This rule is the **commutative property of addition**. We can use the commutative property to rewrite expressions.

Examples:  $2 + 3 = \boxed{5}$  and  $3 + 2 = \boxed{5}$   
 $a + b = b + a$

Even when we add several numbers or variables in a row, the order of the numbers or variables being added isn't important.

Examples:  $2 + 5 + 1 + 9 = \boxed{17}$  and  $5 + 9 + 1 + 2 = \boxed{17}$   
 $a + b + c + d = c + b + d + a$

Note that the sum  $a + a$  can be written as  $2a$ , and that  $5x$  means  $x + x + x + x + x$ .

Example:  $3c + 4c = 7c$

The left-hand side has 3  $c$ 's plus 4 more  $c$ 's, which equals 7  $c$ 's in total.

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### Subtraction is Not Commutative

Numbers or variables cannot be subtracted in any order. Their difference will be affected. Subtraction is not commutative.

Examples:  $5 - 2 = 3$  but  $2 - 5 = -3$   
 $a - b \neq b - a$

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### Commutative Property of Multiplication

Numbers or variables can be multiplied in any order without affecting their product. This rule is the **commutative property of multiplication**. We can use the commutative property of multiplication to rewrite expressions.

Examples:  $2 \times 8 = \boxed{16}$  and  $8 \times 2 = \boxed{16}$   
 $a \times b = b \times a$   
 $3 \times 9 \times 2 \times 10 = 9 \times 3 \times 10 \times 2$   
 $a \times b \times c = c \times b \times a$

*Note:*

- $a \times b$  is usually written as  $ab$  (and  $a \times b \times c$  as  $abc$ ), so we can say  $ab = ba$  and  $abc = cba$ .
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### Division is Not Commutative

Numbers or variables cannot be divided in any order. Their quotient will be affected. Division is not commutative.

Examples:  $\frac{4}{5} = 0.8$  but  $\frac{5}{4} = 1.25$

$$\frac{3}{8} \neq \frac{8}{3}$$

$$\frac{a}{b} \neq \frac{b}{a}$$

*Note:*

- The fractions above are called **reciprocals** of each other.

$$\frac{a}{b} \text{ is the reciprocal of } \frac{b}{a}$$

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### Associative Property of Addition

When adding numbers or variables, we can group them in any way. The **associative property of addition** states that how you group the numbers being added will not affect their sum.

Examples:

$$(3 + 2) + 4 = 5 + 4 = \boxed{9} \quad \text{and} \quad 3 + (2 + 4) = 3 + 6 = \boxed{9}$$

$$(a + b) + c = a + (b + c) = a + b + c$$

$$2 + (3 + 5) + 6 = 2 + 8 + 6 = \boxed{16} \quad \text{and} \quad (2 + 3) + (5 + 6) = 5 + 11 = \boxed{16}$$

$$(a + b) + (c + d) = a + (b + c) + d = a + b + c + d$$

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### Associative Property of Multiplication

When multiplying a series of numbers or variables, we can group them in any way. This rule is the **associative property of multiplication**.

Examples:

$$3 \times (5 \times 2) = \boxed{30} \quad \text{and} \quad (3 \times 5) \times 2 = \boxed{30}$$

$$(4 \times 3) \times 7 = 4 \times (3 \times 7)$$

$$a(bc) = (ab)c$$

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## Distributive Property

Consider solving  $3 \times (5 + 2)$ . You can take two approaches:

1. Add the numbers in parentheses, and then multiply the sum by 3.  
So:  $3 \times 7 = 21$
2. “Distribute” the first factor, 3, through the sum  $(5 + 2)$ . In other words, compute  $3 \times 5$ , then  $3 \times 2$ , and then add the two products:

$$3 \times (5 + 2) = (3 \times 5) + (3 \times 2) = 15 + 6, \text{ which equals } 21.$$

Now consider  $a \times (b + c)$ . Can we use the first approach from above?

Because we don't know the values for  $a$ ,  $b$ , and  $c$ , we cannot simplify  $b + c$ .

We can use the second approach, however:

$$a \times (b + c) = (a \times b) + (a \times c) = ab + ac.$$

Multiplying  $a$  by  $b$  and then by  $c$  is an example of the **distributive property**. This property states that for all numbers  $a$ ,  $b$ , and  $c$ , it is always true that

$$a(b + c) = ab + ac.$$

Examples:

$$2(w + z) = 2w + 2z$$

$$d(h + g) = dh + dg$$

An animation may help you visualize this concept—see the animation in the course online.

Note that this property also applies when the variables are subtracted:

$$a \times (b - c) = ab - ac.$$

Examples:

$$b(s - v) = bs - bv$$

$$3(4 - 7) = 3 \times 4 - 3 \times 7$$

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## **Properties for 0 and 1**

0 and 1 have some unique properties:

$2 + 0 = 2$ $a + 0 = a$	Adding zero doesn't change the number or variable. For any real number $a$ , it's true that $a + 0 = a$ . We say that 0 is the <b>additive identity</b> for the real numbers. The number zero is unique in this respect.
$2 - 0 = 2$ $a - 0 = a$	Subtracting zero also has no effect.
$2 \times 0 = 0$ $a \times 0 = 0$	Multiplying by zero equals zero.
$\frac{2}{0}$ is undefined $\frac{a}{0}$ is undefined	A number cannot be divided by zero.
$\frac{0}{2} = 0$ $\frac{0}{a} = 0$ for $a \neq 0$	Zero divided by any number other than zero is zero.
$2 \times 1 = 2$ $a \times 1 = a$	Multiplying by 1 doesn't change the number or variable. For any real number $a$ , it's true that $a \times 1 = a$ . We say that 1 is the <b>multiplicative identity</b> . The number 1 is unique in this respect.
$\frac{2}{1} = 2$ $\frac{a}{1} = a$	Dividing by 1 doesn't change the number or variable.

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**End of Lesson**