Chapter 2 Section 5 Lesson
Operations with Fractions and Variables

Introduction

This lesson focuses on multiplying, dividing, adding, and subtracting fractions.

Multiplying Fractions

To multiply fractions, multiply the numerators together and multiply the denominators together. The denominators don't have to be the same. Use the same rules you use to multiply signed numbers to multiply signed fractions .

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Note: a , b , c , and d represent any numbers, except that neither b nor d can be zero.

Examples: $1/2 \times 1/4 = 1/8$

$-6/7 \times \left(-3/4\right) = 18/28 = 9/14$ *Note:* we simplified the answer.

Remember to **simplify** an answer, or write it in **lowest terms**, by canceling any factors common to both the numerator and the denominator. In fact, since you have to cancel common factors anyway, it's better to do so *before* you multiply.

Example: Consider the product $36/11 \cdot 33/36$. If you don't cancel common factors before multiplying, you'll have to write $1188/396$ in lowest terms, since $\frac{36}{11} \cdot \frac{33}{36} = \frac{36 \cdot 33}{11 \cdot 36} = \frac{1188}{396}$. Notice that it's easier to start by canceling common factors rather than by multiplying:

$$\frac{36}{11} \cdot \frac{33}{36} = \frac{36 \cdot 33}{11 \cdot 36} = \frac{\cancel{36} \cdot 3 \cdot 11}{11 \cdot \cancel{36}} = \frac{3 \cdot 11}{11} = \frac{3 \cdot \cancel{11}}{\cancel{11}} = 3.$$

Extended Example 1

Multiply: $\frac{16}{111} \cdot \frac{777}{64}$.

Hint: Find the factors that are common to both the numerator and denominator, and combine all the factors into one fraction.

Step 1:

$$\frac{16}{111} \cdot \frac{777}{64} = \frac{4 \cdot 4}{3 \cdot 37} \cdot \frac{21 \cdot 37}{8 \cdot 8} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 37} \cdot \frac{3 \cdot 7 \cdot 37}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$

$$= \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 \cdot 37}{3 \cdot 37 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$

Hint*:* Cancel the factors common to the numerator and the denominator.

Step 2:

$$= \frac{\cancel{2 \cdot 2 \cdot 2 \cdot 2} \cdot \cancel{3} \cdot 7 \cdot \cancel{37}}{\cancel{3} \cdot \cancel{37} \cdot \cancel{2 \cdot 2 \cdot 2 \cdot 2} \cdot 2 \cdot 2} = \frac{7}{2 \cdot 2}$$

Hint*: Now* multiply!

Answer:

$$= \boxed{\frac{7}{4}}$$

Extended Example 2

Multiply: $\frac{-5a}{7} \cdot \frac{3}{4b}$.

Hint: Multiply the numerators together and multiply the denominators together.

Step 1:

$$= \frac{-5a \cdot 3}{7 \cdot 4b}$$

Hint: Handle the numbers and variables separately: multiply the numbers, and the variables "come along for the ride."

Answer:

$$= \frac{-15a}{28b} \text{ or } -\frac{15a}{28b}$$

Dividing Fractions

To divide fractions, multiply the first fraction by the reciprocal of the second fraction (in other words, flip the second fraction's numerator and denominator).

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \quad \text{or} \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Note: Again, no denominator can be zero.

Examples:

$$\frac{1}{3} \div \frac{2}{5} = \frac{1}{3} \times \frac{5}{2}$$ Change ÷ to × and use the second fraction's reciprocal.

$$= \frac{5}{6}$$ Multiply the fractions.

$$\frac{-3}{4} \div \frac{6}{-7} = \frac{-3}{4} \times \frac{-7}{6}$$ Change ÷ to × and use the second fraction's reciprocal.

$$= \frac{21}{24} = \frac{7}{8}$$ Multiply the fractions and simplify the answer.

Negative Fractions

Note that negative fractions can be written in several ways:

$$\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$$

Extended Example 3

Divide: $\dfrac{\frac{-3}{5a}}{\frac{7b}{4}}$.

Hint: It might help to rewrite the problem using the ÷ sign.

Step 1:

$$= \frac{-3}{5a} \div \frac{7b}{4}$$

Hint: Remember the rule: multiply by the reciprocal of the second fraction.

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Step 2:

$$= \frac{-3}{5a} \times \frac{4}{7b}$$

Hint: Multiply the numerators together and the denominators together.

Answer:

$$= -\frac{12}{35ab}$$

Adding and Subtracting Fractions with Like Denominators

If fractions have the same denominators (also called **like denominators**), you can add or subtract the numerators. The denominator stays the same.

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

Study the following examples involving fractions with the *same denominators*:

$\frac{1}{3} + \frac{1}{3} = \frac{1+1}{3} = \frac{2}{3}$		$\frac{a}{b} + \frac{a}{b} = \frac{2a}{b}$
$\frac{-1}{3} + \frac{-1}{3} = \frac{-1+(-1)}{3} = \frac{-2}{3}$		$\frac{-a}{b} + \frac{-a}{b} = -\frac{2a}{b}$
$\frac{1}{3} + \left(-\frac{1}{3}\right) = \frac{1+(-1)}{3} = \frac{0}{3} = 0$		$\frac{a}{b} + \frac{-a}{b} = \frac{0}{b} = 0$
$\frac{3}{4} - \left(\frac{1}{4}\right) = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$		$\frac{3a}{b} - \left(\frac{a}{b}\right) = \frac{2a}{b}$
$\frac{1}{5} - \left(\frac{-2}{5}\right) = \frac{1-(-2)}{5} = \frac{3}{5}$		$\frac{a}{b} - \left(-\frac{2a}{b}\right) = \frac{3a}{b}$

Adding and Subtracting Fractions with Unlike Denominators

To add or subtract fractions with denominators that are not the same, we must first find a **common denominator**. Then, we can rewrite the fractions using that common denominator and add them just as we would add any fractions with like denominators. This is shown in the following examples. Pay attention to the signs as you review the examples.

Example A

$\frac{1}{3}+\frac{1}{2}=?$

These fractions have unlike denominators. We need to find a common denominator in order to add them together. We know that 3 and 2 both divide into 6 evenly, so 6 is a common multiple of 3 and 2; in other words, 6 is a common denominator of $1/3$ and $1/2$. First, we need to rewrite $1/3$ with a 6 in the denominator, so we have to multiply the 3 in the denominator by 2. Therefore we will multiply $\frac{1}{3}$ by $\frac{2}{2}$: $\frac{1}{3}\times\frac{2}{2}=\frac{5}{6}$.

Notice that we multiplied $1/3$ by $2/2$, which is 1, the multiplicative identity, so we did not change the value of $1/3$. Next, we need to rewrite $1/2$ with a 6 in the denominator. We'll need to multiply its denominator by 3 to get 6, so we'll multiply the fraction by the multiplicative identity $3/3$, which won't change the fraction's value. The steps are combined here:

$$\frac{1}{3}+\frac{1}{2}=\left(\frac{1}{3}\times\frac{2}{2}\right)+\left(\frac{1}{2}\times\frac{3}{3}\right)=\frac{2}{6}+\frac{3}{6}=\frac{2+3}{6}=\frac{5}{6}$$

Example B

$-\frac{1}{6}+\left(-\frac{5}{12}\right)=?$

$$-\frac{1}{6}+\left(-\frac{5}{12}\right)=\left(-\frac{1}{6}\right)\times\frac{2}{2}+\left(-\frac{5}{12}\right)$$ The common denominator is 12.

$$=\left(-\frac{2}{12}\right)+\left(-\frac{5}{12}\right)$$

$$=\frac{-2+(-5)}{12}=\frac{-7}{12}$$, which we can write as $-\frac{7}{12}$.

Example C

$\frac{3}{10}-\frac{4}{25}=?$

$$\frac{3}{10}-\frac{4}{25}=\frac{3}{10}\times\frac{5}{5}-\left(\frac{4}{25}\times\frac{2}{2}\right)$$ The common denominator is 50.

$$=\frac{15}{50}-\frac{8}{50}=\frac{7}{50}$$

Example D

$\frac{11}{40}+\left(\frac{-1}{8}\right)=?$

$$\frac{11}{40}+\left(\frac{-1}{8}\right)=\frac{11}{40}+\left(-\frac{1}{8}\right)\left(\frac{5}{5}\right)$$ The common denominator is 40.

$$=\frac{11}{40}+\left(\frac{-5}{40}\right)$$

$$=\frac{11+(-5)}{40}$$

$$=\frac{6}{40}=\frac{3}{20}$$ Simplify to find the final answer.

Example E

$\frac{-4}{15}+\frac{-3}{20}=?$

$$\frac{-4}{15}+\frac{-3}{20}=\frac{-4}{15}\left(\frac{4}{4}\right)+-\frac{3}{20}\left(\frac{3}{3}\right)$$ The common denominator is 60.

$$=\frac{-16}{60}+\frac{-9}{60}$$

$$=\frac{-16+-9}{60}$$

$$=-\frac{25}{60}=-\frac{5}{12}$$ Simplify to find the final answer.

Extended Example 4

Add: $\frac{3}{7}+\frac{2}{5}$.

Hint : Find a common denominator.

Step 1:

7 and 5 both divide evenly into 35.

Hint: Rewrite the problem with 35 as a common denominator.

Step 2:

$$\frac{3}{7}+\frac{2}{5}=\frac{3}{7}\left(\frac{5}{5}\right)+\frac{2}{5}\left(\frac{7}{7}\right)=\frac{15}{35}+\frac{14}{35}$$

Hint: Add the numerators.

Answer:

$$=\frac{29}{35}$$

End of Lesson