


## Chapter 2 Section 7 Lesson Order of Operations

### Introduction

Without rules to follow, we might end up with several answers for the same problem. To avoid this, we follow a set of rules called the **order of operations**. This lesson reviews this set of rules.

---

Simplify:  
 $12 + 3 \times 2$



If we **add** first, we get:  $15 \times 2 = 30$       If we **multiply** first, we get:  $12 + 6 = 18$

**Which one is correct?**

**Rule 1:** Always multiply and divide (from left to right) before adding and subtracting (from left to right).

Rule 1 lets us solve the equation above in only one way—by multiplying first:

$$12 + 3 \times 2 = 12 + 6 = 18$$

Additional examples:

Simplify:  $24 \div 4 + 4 \times 2$ .

$$\begin{aligned} 24 \div 4 + 4 \times 2 &= 6 + 4 \times 2 \\ &= 6 + 8 \\ &= 14 \end{aligned}$$


Simplify:  $3 \times 4 + 16 \div 2 - 3$ .

$$\begin{aligned} 3 \times 4 + 16 \div 2 - 3 &= 12 + 16 \div 2 - 3 \\ &= 12 + 8 - 3 \\ &= 20 - 3 \\ &= 17 \end{aligned}$$

---

---

Simplify:  
 $40 \div 5 \times 4$

If we divide first, we get:  If we multiply first, we get:

$8 \times 4 = 32$                        $40 \div 20 = 2$

**Which one is correct?**

Rule 1 from the previous screen requires that multiplication or division be performed as encountered from left to right. Division is the first operation we should perform, so the answer is 32 :

$$40 \div 5 \times 4 = 8 \times 4 = 32 .$$

Example: Simplify  $100 \div 25 \times 4 + 6(-4) - 19$ .

$$\begin{aligned} 100 \div 25 \times 4 + 6(-4) - 19 &= 4 \times 4 + 6(-4) - 19 && \text{Divide } 100 \div 25 = 4 \\ &= 16 + 6(-4) - 19 && \text{Multiply } 4 \times 4 = 16 \\ &= 16 - 24 - 19 && \text{Then, multiply } 6(-4) = -24 \\ &= -8 - 19 && \text{Then, subtract } 16 - 24 = -8 \\ &= -27 && \text{Subtract.} \end{aligned}$$

---

### Grouping Symbols

**Grouping symbols** include parentheses, ( ), brackets, [ ], or braces, { }. There really is no difference between one type of grouping symbol and another. Just remember that a left grouping symbol, such as (, is always paired with a right grouping symbol—in this case, ). Sometimes various grouping symbols are used in the same problem to help you see which left grouping symbol goes with which right grouping symbol.

Two additional symbols that function as grouping symbols are the radical sign,  $\sqrt{\quad}$ , and the fraction bar:  $\frac{\quad}{\quad}$ . You've encountered both symbols already, but you might not have thought of them as grouping symbols.

Grouping symbols are sometimes used only to eliminate ambiguity. They are usually used, however, to force expressions to be evaluated in a way that would otherwise violate the order of operation conventions.

Examples:

$$3 \times 5 + 8 = 15 + 8 \quad \text{but} \quad 3 \times (5 + 8) = 3 \times 13$$
$$= 23 \quad \quad \quad = 39$$

$$5 + 2 \times 10 - 4 = 5 + 20 - 4 \quad \text{but} \quad 5 + 2 \times (10 - 4) = 5 + 2 \times 6$$
$$= 25 - 4 \quad \quad \quad = 5 + 12$$
$$= 21 \quad \quad \quad = 17$$

---

The rules for the order of operations specify when to evaluate expressions within grouping symbols.

**Rule 2:** Perform operations within parentheses and other grouping symbols first. Then follow Rule 1.

---

**Example A**

Simplify  $\frac{15-9}{3}$ .

Simplify within the grouping symbol (the fraction bar) first:

$$\begin{aligned} \frac{15-9}{3} &= \frac{6}{3} \\ \text{Then, divide.} \quad &= 6 \div 3 \\ &= 2 \end{aligned}$$

The fraction bar separates the numerator from the denominator. The numerator and denominator have to be simplified before we can divide.

---

**Example B**

Simplify  $5 + 4\sqrt{17 + 83}$ .

Simplify within the grouping symbol, the radical sign, first.

$$\begin{aligned} 5 + 4\sqrt{17 + 83} &= 5 + 4\sqrt{100} && \text{Note: } \sqrt{100} = 10 \text{ because } 10 \cdot 10 = 100 \\ &= 5 + 4(10) \end{aligned}$$

$$\begin{aligned} \text{Then, multiply.} \quad &= 5 + 40 \\ &= 45 \end{aligned}$$

---

When layers of grouping symbols are nested within each other, it helps to have a rule to guide us.

**Rule 3:** When an expression contains more than one grouping symbol, perform the operations within the innermost grouping symbol first. Then work toward the outermost grouping symbol.

---

**Example C**

Simplify  $2[4(3 + 2) - 5]$ .

$$\begin{aligned}2[4(3 + 2) - 5] &= 2[4(5) - 5] \quad \text{Simplify within innermost grouping symbol first.} \\ &= 2[20 - 5] \quad \text{Then, simplify within the brackets.} \\ &= 2 \cdot 15 \\ &= 30\end{aligned}$$

*Note:* Brackets, [ and ], are sometimes used as outer grouping symbols when parentheses are nested inside them.

---

**Example D**

Simplify  $40[18 - (15 - 48) - 12]$ .

$$\begin{aligned}40[18 - (15 - 48) - 12] &= 40[18 - (-33) - 12] \\ &= 40[18 + 33 - 12] \\ &= 40[51 - 12] \\ &= 40 \cdot 39 \\ &= 1,560\end{aligned}$$

---

**Extended Example 1a**

Simplify  $(2 + 5) \times 6 \div 3 + 3$ .

Hint: Perform operations within grouping symbols first.

*Step 1:*

$$= 7 \times 6 \div 3 + 3$$

Hint: Multiply and divide before adding and subtracting, from left to right.

*Answer:*

$$\begin{aligned}&= 42 \div 3 + 3 \\ &= 14 + 3 \\ &= 17\end{aligned}$$

---

---

**Extended Example 1b**

Simplify  $3[2 - 3(5 \times 2) + 4]$ .

Hint: Simplify within innermost grouping symbol first.

*Step 1:*

$$= 3[2 - 3(10) + 4]$$

Hint: Then, simplify within the nested grouping symbol. Within the brackets, multiply and divide before adding and subtracting from left to right.

*Answer:*

$$= 3[2 - 30 + 4]$$

$$= 3[-24]$$

$$= -72$$

---

**End of Lesson**