Chapter 3 Section 1 Lesson
Monomials

Introduction

This lesson introduces monomials, exponents, and associated terminology.

Definitions

Like any other subject, algebra has its own vocabulary— sets of words that are specific to the subject. To understand algebra, you must learn its vocabulary.

A **variable** is a letter or symbol used to represent a quantity that is unknown or can change. The letters x and y are the symbols most commonly used as variables but any letter can be used. Variables are also sometimes referred to as "unknowns." Common nouns can serve this purpose in the English language. For example, the word “cat” represents different cats; the variable x represents different numbers.

A **constant** is a quantity that does not change in value. For example, 3, -8, $\frac{3}{7}$, 36.5, $\sqrt{2}$, and π are all constants.

A **monomial** is a constant, a variable, or the product of constants and variables. A monomial never involves addition, subtraction, radicals of variables, or variables in a denominator.
For example, 2, $9xy$, $-4u^7v^5w^{13}$, and $\frac{1}{2}a^2b$ are all monomials. The following are <u>not</u> monomials: $9x+2y$, $2\sqrt{x}$, $\frac{5}{x}$, $\frac{1}{3\sqrt{x}}$, and $\frac{5}{3x^4}$.

Based on the descriptions above, try to answer the following questions yourself before looking at the answers.

Question: Is $7b$ a monomial?

> *Answer:* Yes, because 7 is a constant, b is a variable, and $7b$ is their product.

Question: Is $7b+2x$ a monomial?

> *Answer:* No. It is the sum of two monomials.

Question: Is $\sqrt{2xy}$ a monomial?

> *Answer:* No. Monomials can never have variables under the radical sign.

Example A

Is $7b(2x)$ a monomial?

In this case, $7b$ and $2x$ are factors to be multiplied: $7b(2x) = 14bx$.
(Remember, when you multiply, the variables "go along for the ride.")
The result is the product of the constant 14 and the variables b and x.
So it is a monomial.

Notes:

- The monomial $14bx$ is written in **standard form**. This means that the constant comes first and the variables come second, in alphabetical order, when writing the product. In the question above, $7b(2x)$ is a monomial, but it is not in standard form.

- When a monomial is written in standard form, the constant is called the **coefficient** of the monomial. In the monomial $14bx$, 14 is the coefficient.

Try to answer the following questions yourself before looking at the answers.

Question: Write $5m \cdot 2q$ in standard form and find the coefficient.

Answer: The standard form is $10mq$, and the coefficient of the monomial is 10.

Question: Write $4n(5x)(2c)$ in standard form, and find the coefficient.

Answer: The standard form is $40cnx$, and the coefficient is 40.

Question: What is the coefficient of $-xyz$?

Answer: Although no number appears in the monomial, the coefficient is -1 because $-xyz = -1(xyz)$.

Question: Write $-3x(-2y)(-z)$ in standard form, and find the coefficient.

Answer: Multiply the numbers $(-3)(-2)(-1)$.
The variables go along for the ride: $-6xyz$. The coefficient is -6.

Exponential Notation

Products of monomials with repeating factors such as $x \cdot 3 \cdot x$ are not written as $3xx$. Instead, a special notation is used. The product of $x \cdot x$ is written as x^2, and read as "x to the second power." So, $x \cdot 3 \cdot x = 3 \cdot x \cdot x = 3x^2$, indicating that x is used as a factor two times.

The repeated factor, x, is called the **base**.

The number of repeated factors, 2, is called the **exponent**.

Note:

- The expression x^2 can also be read as "x squared," and the expression x^3 can be read as "x to the third power" or "x cubed."

Examples:

$x \cdot x \cdot x \cdot x \cdot x = x^5$ and is read "x to the fifth power"

$y \cdot y \cdot y = y^3$ and is read "y to the third power"

Consider the three ways of writing 3^4: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

- 3^4 is said to be in **exponential form**
- $3 \cdot 3 \cdot 3 \cdot 3$ is in **expanded form**
- 81 is in **standard form**

Question: Write 625 in exponential form, as a power of 5.

Answer: Ask yourself: "Raising 5 to what power equals 625?"
$625 = 5 \cdot 5 \cdot 5 \cdot 5$, so there are four factors of 5: $625 = 5^4$.

Degree

The **degree** of a monomial with only one variable is simply the degree of that variable.

Examples:

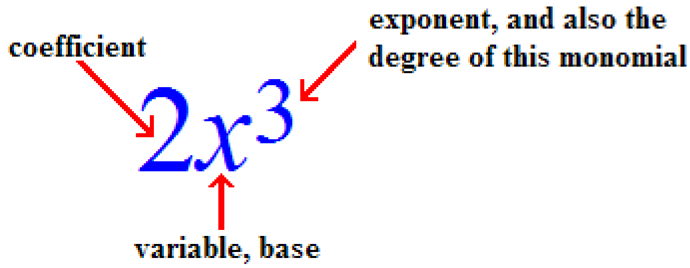
The degree of $5x^4$ is 4.

The degree of $32y^9$ is 9.

The degree of x is 1 since $x = x^1$.

The degree of a constant is 0.

The following illustration may help you understand some of the terminology we've just studied:



Example B

What is the standard form of $x^2 \cdot 4 \cdot x$, and what is this monomial's degree?

$$\begin{aligned} x^2 \cdot 4 \cdot x &= 4 \cdot x^2 \cdot x \\ &= 4 \cdot x \cdot x \cdot x \\ &= 4x^3 \end{aligned}$$

The standard form is $4x^3$. The degree of $4x^3$ is 3, since x is used as a factor three times.

Example C

Write the monomial $x \cdot 7 \cdot x \cdot 3$ in standard form, and identify the coefficient, variable, and degree.

Standard Form	Coefficient	Variable	Degree
$x \cdot 7 \cdot x \cdot 3 = 21x^2$	21	x	2

Example D

Write the monomial $4 \cdot y \cdot 5 \cdot y \cdot y$ in standard form, and identify the coefficient, variable, and degree.

Standard Form	Coefficient	Variable	Degree
$4 \cdot y \cdot 5 \cdot y \cdot y = 20y^3$	20	y	3

Question: Write the monomial $5a^2 \cdot 3a^3$ in standard form; state its coefficient and degree.

Answer: Multiply the coefficients: $5 \cdot 3 = 15$, which is the coefficient of the monomial in standard form. Now find the number of factors of a: there are $2 + 3 = 5$ factors of a. So $5a^2 \cdot 3a^3 = 15a^5$, with coefficient 15 and degree 5.

Example E

Write the monomial $6 \cdot x^3 \cdot y \cdot 4 \cdot xy^2$ in standard form.

It might help to reorganize the expression by putting the numbers and variables next to each other and in alphabetical order.

$$\begin{aligned} 6 \cdot x^3 \cdot y \cdot 4 \cdot xy^2 &= 6 \cdot 4 \cdot x^3 \cdot x \cdot y \cdot y^2 \\ &= 24x^4y^3 \end{aligned}$$

Since x is used as a factor 4 times and y is used as a factor 3 times, $24x^4y^3$ is the standard form of the given monomial.

Example F

Write the product of $7x^3y^2$, $3xyz$, and $4yz^4$ as a monomial in standard form.

Since $7 \cdot 3 \cdot 4 = 84$, the coefficient of the product is 84. There are 4 factors of x, 4 factors of y, and 5 factors of z, so: $7x^3y^2 \cdot 3xyz \cdot 4yz^4 = 84x^4y^4z^5$.

The monomial $84x^4y^4z^5$ is in standard form.

Example G

Write the product of the monomials $2a^2c^2$, $6ab$, $2bc^3$, and abc as a monomial in standard form. (Remember that 1 is the coefficient of abc).

There are 4 factors of a, 3 factors of b, and 6 factors of c. The coefficient is $2 \cdot 6 \cdot 2 \cdot 1 = 24$.

$$2a^2c^2 \cdot 6ab \cdot 2bc^3 \cdot abc = 24a^4b^3c^6$$

Example H

Write the product of the monomials 2^3xy, $-10x^2z$, and wxz in standard form.

$$\begin{aligned} 2^3xy\left(-10x^2z\right)wxz &= 8 \cdot \left(-10\right) \cdot w \cdot x \cdot x^2 \cdot x \cdot y \cdot z \cdot z \\ &= -80wx^4yz^2 \end{aligned}$$

Notes:

- When a product involves negative monomials, use parentheses to help keep things clear when multiplying.
- The sign of a monomial in standard form is the sign of its coefficient.

Extended Example 1a
Write $3x^2y \cdot \left(-2x^2y^3z\right) \cdot 4xz^2$ in standard form.

Hint: Multiply the coefficients.

Step 1:
$3 \cdot (-2) \cdot 4 = -24$

Hint: Put all the variables into alphabetical order, to the right of the coefficient you just found.

Step 2:
$= -24x^2x^2x\,yy^3zz^2$

Hint: Find the number of factors of x, y, and z. Then, write the product in standard form.

Answer:
There are 5 factors of x, 4 factors of y, and 3 factors of z.
The standard form is $-24x^5y^4z^3$.
That is: $3x^2y \cdot \left(-2x^2y^3z\right) \cdot 4xz^2 = -24x^5y^4z^3$.

Extended Example 1b
Write $2x^3y^2 \cdot \left(-4xy^2z^2\right) \cdot \left(-5x^2z^3\right)$ in standard form.

Hint: Multiply the coefficients.

Step 1:
$2 \cdot (-4) \cdot (-5) = 40$

Hint: Put all the variables into alphabetical order, to the right of the coefficient.

Step 2:
$= 40x^3xx^2\,y^2y^2z^2z^3$

Hint: Find the number of factors of x, y, and z. Then, write the product in standard form.

Answer:
There are 6 factors of x, 4 factors of y, and 5 factors of z.
The standard form is $40x^6\,y^4z^5$.
That is, $2x^3y^2 \cdot \left(-4xy^2z^2\right) \cdot \left(-5x^2z^3\right) = 40x^6\,y^4z^5$.

Extended Example 1c

Write $\left(-3u^4v^3w\right)\cdot\left(-5uv^2\right)\cdot\left(-7u^3vw^3\right)$ in standard form.

Hint: Multiply the coefficients.

Step 1:

$(-3)\cdot(-5)\cdot(-7) = -105$

Hint: Put all the variables into alphabetical order, to the right of the coefficient.

Step 2:

$= -105u^4uu^3v^3v^2vww^3$

Hint: Find the number of factors of u, v, and w. Then, write the product in standard form.

Answer:

There are 8 factors of u, 6 factors of v, and 4 factors of w.

The standard form is $-105u^8v^6w^4$.

That is, $\left(-3u^4v^3w\right)\cdot\left(-5uv^2\right)\cdot\left(-7u^3vw^3\right) = -105u^8v^6w^4$.

Extended Example 2a

Write $\left(-4A^2\right)\left(-5B^3\right)(AB)$ in standard form and note the coefficient.

Hint: Multiply the coefficients $(-4)(-5)(1)$.

Step 1:

$(-4)(-5)(1) = 20$

Hint: Put all the variables into alphabetical order, to the right of the coefficient you just found.

Step 2:

$= 20A^2AB^3B$

Hint: Find the number of factors of A and B and write the product in standard form.

Answer:

$= 20A^3B^4$

The coefficient is 20.

Extended Example 2b

Write $\left(A^3\right)\left(-9B\right)\left(2A^2B^4\right)$ in standard form, and note the coefficient.

Hint: Multiply the coefficients.

Step 1:

$1\cdot\left(-9\right)\cdot 2=-18$

Hint: Put all the variables into alphabetical order, to the right of the coefficient.

Step 2:

$=-18A^3A^2BB^4$

Hint: Find the number of factors of each variable and write the product in standard form.

Answer:

$=-18A^5B^5$

The coefficient is -18.

Extended Example 2c

Write $\left(-3c^2d^2\right)\left(cd\right)\left(-10c^3d^4\right)$ in standard form, and note the coefficient.

Hint: Multiply the coefficients.

Step 1:

$\left(-3\right)\left(1\right)\left(-10\right)=30$

Hint: Put all the variables into alphabetical order, to the right of the coefficient.

Step 2:

$=30c^2cc^3d^2dd^4$

Hint: Find the number of factors of each variable and write the product in standard form.

Answer:

$=30c^6d^7$

The coefficient is 30.

Extended Example 3a

Write $\left(-5u^2v^5w^3\right)\left(-20u^4v^3w^4\right)$ in standard form and note the coefficient.

Hint: Multiply the coefficients.

Step 1:

$(-5)(-20)=100$

Hint: Put all the variables into alphabetical order, to the right of the coefficient.

Step 2:

$=100u^2u^4v^5v^3w^3w^4$

Hint: Find the number of factors of u, v, and w, and write the product in standard form.

Answer:

$=100u^6v^8w^7$

The coefficient is 100.

Extended Example 3b

Write $\left(-3r^4s^3t^2\right)\left(7r^2s^2t^3\right)$ in standard form, and note the coefficient.

Hint: Multiply the coefficients.

Step 1:

$(-3)(7)=-21$

Hint: Put all the variables into alphabetical order, to the right of the coefficient.

Step 2:

$=-21r^4r^2s^3s^2t^2t^3$

Hint: Find the number of factors of each variable and write the product in standard form.

Answer:

$=-21r^6s^5t^5$

The coefficient is -21.

Extended Example 3c

Write $\left(-8r^3s^4t^6\right)\left(-9r^5s^2t^3\right)$ in standard form, and note the coefficient.

Hint: Multiply the coefficients.

Step 1:

$(-9)(-8) = 72$

Hint: Put all the variables into alphabetical order, to the right of the coefficient.

Step 2:

$= 72r^3r^5s^4s^2t^6t^3$

Hint: Find the number of factors of each variable and write the product in standard form.

Answer:

$= 72r^8s^6t^9$

The coefficient is 72.

Extended Example 4a

Write $\left(-2x^2\right)\left(-3x^4\right)(7x)\left(-2x^6\right)$ in standard form. Note the coefficient and the degree.

Hint: Multiply the coefficients.

Step 1:

$(-2)(-3)(7)(-2) = -84$

Hint: Put the variables to the right of the coefficient.

Step 2:

$= -84x^2x^4xx^6$

Hint: Find the number of factors of x and write the product in standard form.

Answer:

$= -84\,x^{2+4+1+6} = -84\,x^{13}$

The coefficient is -84, and the degree is 13.

Extended Example 4b

Write $\left(3x^7\right)\left(-9x^5\right)\left(-2x^3\right)(-5x)$ in standard form. Note the coefficient and the degree.

Hint: Multiply the coefficients.

Step 1:

$(3)(-9)(-2)(-5) = -270$

Hint: Put the variables to the right of the coefficient.

Step 2:

$= -270\,x^7x^5x^3x$

Hint: Find the number of factors of x and write the product in standard form.

Answer:

$= -270\,x^{7+5+3+1}$

$= -270\,x^{16}$

The coefficient is -270, and the degree is 16.

Extended Example 4c

Write $\left(-4y^3\right)\left(6y^6\right)(3y)\left(-2y^4\right)$ in standard form. Note the coefficient and the degree.

Hint: Multiply the coefficients.

Step 1:

$(-4)(6)(3)(-2) = 144$

Hint: Put the variables to the right of the coefficient.

Step 2:

$= 144\,y^3y^6yy^4$

Hint: Find the number of factors of y and write the product in standard form.

Answer:

$= 144y^{3+6+1+4}$

$= 144\,y^{14}$

The coefficient is 144, and the degree is 14.

Extended Example 5a
Simplify $\left(3a^4b^2\right)\left(-10a^2b^{11}\right)$. Write the answer in standard form.

Hint: Multiply the coefficients.

Step 1:
$3\cdot(-10) = -30$

Hint: Put the variables to the right of the coefficient.

Step 2:
$= -30\cdot a^4\cdot a^2\cdot b^2\cdot b^{11}$

Hint: Find the number of factors of a and b and write the product in standard form.

Answer:
$= -30a^6b^{13}$

Extended Example 5b
Simplify $\left(-11a^5b^7\right)\left(9a^6b^5\right)$. Write the answer in standard form.

Hint: Multiply the coefficients.

Step 1:
$(-11)\cdot 9 = -99$

Hint: Put the variables to the right of the coefficient.

Step 2:
$= -99a^5a^6b^7b^5$

Hint: Find the number of factors of each variable and write the product in standard form.

Answer:
$= -99a^{5+6}b^{7+5}$
$= -99a^{11}b^{12}$

Extended Example 5c

Simplify $\left(-100x^{12}y^{10}\right)\left(-17x^{20}y^{13}\right)$. Write the answer in standard form.

Hint: Multiply the coefficients.

Step 1:

$(-100)\cdot(-17)=1700$

Hint: Put the variables to the right of the coefficient.

Step 2:

$=1700x^{12+20}y^{10+13}$

Hint: Find the number of factors of each variable and write the product in standard form.

Answer:

$=1700x^{32}y^{23}$

End of Lesson