Chapter 3 Section 2 Lesson
Basic Rules for Exponents

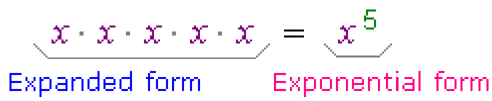
Introduction

It is important to memorize the rules for exponents. Exponents will be used frequently in this course, as well as in the next. Try to understand each rule by carefully examining the examples in this lesson. All of the rules follow from the basic definition of exponents given below.

Definition of Exponents:

$$x^n = \overbrace{x \cdot x \cdot x \cdots x}^{n \text{ of these}}$$

For example, $x^2 = x \cdot x$, $x^3 = x \cdot x \cdot x$, etc.



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As you study the rules of exponents in this lesson, assume all exponents are positive integers.

> **Rule 1:** $x^1 = x$, for any real number x.

Examples: $23^1 = 23$

$A^1 = A$

> **Rule 2:** $x^m \cdot x^n = x^{m+n}$, for any real number x and any integers m and n.

Examples: $5^4 \cdot 5^3 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^7$

or: $5^4 \cdot 5^3 = 5^{4+3} = 5^7$

$x^7 \cdot x^8 = x^{7+8} = x^{15}$

$3^5 \cdot 3^2 = 3^{5+2} = 3^7$

> **Rule 3:** $(xy)^n = x^n \cdot y^n$, for any real numbers x and y and any positive integer n.

Examples:

$(3x)^4 = 3x \cdot 3x \cdot 3x \cdot 3x = 3 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x = 3^4 x^4 = 81x^4$

or: $(3x)^4 = 3^4 x^4 = 81x^4$

$(2ab)^3 = 2^3 a^3 b^3 = 8a^3 b^3$

Rule 4: $\left(x^m\right)^n = x^{m \cdot n}$, for any real number x and any integers m and n.

Examples:

$$\left(7^3\right)^4 = 7^3 \cdot 7^3 \cdot 7^3 \cdot 7^3 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^{12}$$

or: $\left(7^3\right)^4 = 7^{3 \cdot 4} = 7^{12}$

$$\left(x^2\right)^3 = x^2 \cdot x^2 \cdot x^2 = x^6$$

or: $\left(x^2\right)^3 = x^{2 \cdot 3} = x^6$

Example A

Multiply $m^7 \cdot m$. Write the product as a power with base m.

Use Rule 1, $x^1 = x$, to write $m = m^1$: $m^7 \cdot m^1$

Then use Rule 2, $x^m \cdot x^n = x^{m+n}$: $m^7 \cdot m^1 = m^{7+1} = m^8$.

Question: Multiply $5^2 \cdot 5 \cdot 5^4 \cdot 5^2$. Write the product as a power with base 5.

Answer: Use Rule 1, $x^1 = x$, and Rule 2, $x^m \cdot x^n = x^{m+n}$:

$5^2 \cdot 5 \cdot 5^4 \cdot 5^2 = 5^2 \cdot 5^1 \cdot 5^4 \cdot 5^2 = 5^{2+1+4+2} = 5^9$.

Example B

Write $5^3 x^3$ as a power with base $5x$.

Use Rule 3, $(xy)^n = x^n \cdot y^n$: $5^3 x^3 = (5x)^3$.

Example C

Write 16^3 as a power with base 2.

Use Rule 4, $\left(x^m\right)^n = x^{m \cdot n}$, and the fact that $16 = 2^4$:

$16^3 = \left(2^4\right)^3 = 2^{4 \cdot 3} = 2^{12}$.

Extended Example 1a

Write the monomial $\left(4x^2y^5\right)^3$ in standard form.

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 1:

$= 4^3\left(x^2\right)^3\left(y^5\right)^3$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m\cdot n}$.

Step 2:

$= 4^3x^{2\cdot 3}y^{5\cdot 3}$

Hint: Simplify.

Answer:

$= 64x^6y^{15}$

Extended Example 1b

Write $\left(3x^4y^6\right)^2$ in standard form.

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 1:

$= 3^2\left(x^4\right)^2\left(y^6\right)^2$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m\cdot n}$.

Step 2:

$= 3^2x^{4\cdot 2}y^{6\cdot 2}$

Hint: Simplify.

Answer:

$= 9x^8y^{12}$

Extended Example 1c

Write $\left(5u^6v^9\right)^4$ in standard form.

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 1:

$= 5^4\left(u^6\right)^4\left(v^9\right)^4$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m\cdot n}$.

Step 2:

$= 5^4u^{6\cdot 4}v^{9\cdot 4}$

Hint: Simplify.

Answer:

$= 625u^{24}v^{36}$

Extended Example 2a

Write the monomial $\left(2A^4B^2C^3\right)^5$ in standard form.

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 1:

$= 2^5\left(A^4\right)^5\left(B^2\right)^5\left(C^3\right)^5$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m\cdot n}$.

Step 2:

$= 2^5A^{4\cdot5}B^{2\cdot5}C^{3\cdot5}$

Hint: Simplify.

Answer:

$= 32A^{20}B^{10}C^{15}$

Extended Example 2b

Write the monomial $\left(2a^{10}b^9c^{11}\right)^4$ in standard form.

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 1:

$= 2^4\left(a^{10}\right)^4\left(b^9\right)^4\left(c^{11}\right)^4$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m\cdot n}$.

Step 2:

$= 2^4a^{10\cdot4}b^{9\cdot4}c^{11\cdot4}$

Hint: Simplify.

Answer:

$= 16a^{40}b^{36}c^{44}$

Extended Example 2c

Write the monomial $\left(2r^2s^{30}t^{100}\right)^6$ in standard form.

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 1:

$= 2^6\left(r^2\right)^6\left(s^{30}\right)^6\left(t^{100}\right)^6$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m\cdot n}$.

Step 2:

$= 2^6r^{2\cdot6}s^{30\cdot6}t^{100\cdot6}$

Hint: Simplify.

Answer:

$= 64r^{12}s^{180}t^{600}$

Rule 5: $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$, for any x and y, $y \neq 0$, and any integer n.

Examples:

When you multiply fractions, you multiply the numerators together and the denominators together, so this rule seems reasonable:

$$\left(\frac{2}{3}\right)^4 = \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \frac{2\cdot 2\cdot 2\cdot 2}{3\cdot 3\cdot 3\cdot 3} = \frac{16}{81}$$

or: $$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$$

$$\left(\frac{u}{v}\right)^3 = \frac{u}{v}\cdot\frac{u}{v}\cdot\frac{u}{v} = \frac{u\cdot u\cdot u}{v\cdot v\cdot v} = \frac{u^3}{v^3}$$

or: $$\left(\frac{u}{v}\right)^3 = \frac{u^3}{v^3}$$

Extended Example 3a

Simplify $\left(\frac{3z^2}{t^2}\right)^2$.

Hint: Use Rule 5, $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$.

Step 1:

$$= \frac{\left(3z^2\right)^2}{\left(t^2\right)^2}$$

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$, in the numerator.

Step 2:

$$= \frac{3^2\left(z^2\right)^2}{\left(t^2\right)^2}$$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m\cdot n}$, in the numerator and denominator.

Step 3:

$$= \frac{3^2 z^{2\cdot 2}}{t^{2\cdot 2}}$$

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Hint: Simplify.

Answer:

$$= \frac{9z^4}{t^4}$$

Extended Example 3b

Simplify $\left(\frac{2a^5}{5b^4}\right)^3$.

Hint: Use Rule 5, $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$.

Step 1:

$$= \frac{\left(2a^5\right)^3}{\left(5b^4\right)^3}$$

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$, in the numerator and the denominator.

Step 2:

$$= \frac{2^3\left(a^5\right)^3}{5^3\left(b^4\right)^3}$$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m \cdot n}$, in the numerator and denominator.

Step 3:

$$= \frac{2^3 a^{5 \cdot 3}}{5^3 b^{4 \cdot 3}}$$

Hint: Simplify.

Answer:

$$= \frac{8a^{15}}{125b^{12}}$$

Extended Example 4a

Simplify $\left(\frac{z^2}{2t^3}\right)^3$.

Hint: Use Rule 5, $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$.

Step 1:

$$= \frac{\left(z^2\right)^3}{\left(2t^3\right)^3}$$

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$, in the denominator.

Step 2:

$$= \frac{\left(z^2\right)^3}{2^3\left(t^3\right)^3}$$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m \cdot n}$, in the numerator and denominator.

Step 3:

$$= \frac{z^{2 \cdot 3}}{2^3 t^{3 \cdot 3}}$$

Hint: Simplify.

Answer:

$$= \frac{z^6}{8t^9}$$

Extended Example 4b

Simplify $\left(\frac{x^7}{3y^5}\right)^4$.

Hint: Use Rule 5, $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$.

Step 1:

$$= \frac{\left(x^7\right)^4}{\left(3y^5\right)^4}$$

continued…

continued…

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 2:

$$= \frac{\left(x^7\right)^4}{3^4\left(y^5\right)^4}$$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m \cdot n}$.

Step 3:

$$= \frac{x^{7\cdot 4}}{3^4 y^{5\cdot 4}}$$

Hint: Simplify.

Answer:

$$= \frac{x^{28}}{81y^{20}}$$

Extended Example 5a

Simplify $\left(\frac{2v^4}{3w^5}\right)^3$.

Hint: Use Rule 5, $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$.

Step 1:

$$= \frac{\left(2v^4\right)^3}{\left(3w^5\right)^3}$$

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 2:

$$= \frac{2^3\left(v^4\right)^3}{3^3\left(w^5\right)^3}$$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m \cdot n}$.

Step 3:

$$= \frac{8v^{4\cdot 3}}{27w^{5\cdot 3}}$$

continued…

continued…

Hint: Simplify.

Answer:

$$= \frac{8v^{12}}{27w^{15}}$$

Note:

- You can go directly from Step 1 to the solution once you are familiar with this process.

Extended Example 5b

Simplify $\left(\frac{6a^6}{7b^7}\right)^3$.

Hint: Use Rule 5, $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$.

Step 1:

$$= \frac{\left(6a^6\right)^3}{\left(7b^7\right)^3}$$

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 2:

$$= \frac{6^3\left(a^6\right)^3}{7^3\left(b^7\right)^3}$$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m \cdot n}$.

Step 3:

$$= \frac{6^3 a^{6\cdot 3}}{7^3 b^{7\cdot 3}}$$

Hint: Simplify.

Answer:

$$= \frac{216a^{18}}{343b^{21}}$$

Extended Example 5c

Simplify $\left(\dfrac{3z^{11}}{5y^5}\right)^4$.

Hint: Use Rule 5, $\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$.

Step 1:

$$= \frac{\left(3z^{11}\right)^4}{\left(5y^5\right)^4}$$

Hint: Use Rule 3, $(xy)^n = x^n \cdot y^n$.

Step 2:

$$= \frac{3^4\left(z^{11}\right)^4}{5^4\left(y^5\right)^4}$$

Hint: Use Rule 4, $\left(x^m\right)^n = x^{m \cdot n}$.

Step 3:

$$= \frac{3^4\, z^{11\cdot 4}}{5^4\, y^{5\cdot 4}}$$

Hint: Simplify.

Answer:

$$= \frac{81z^{44}}{625\,y^{20}}$$

Consider the expression $3 \cdot 2^5$.

It looks like there are two ways to calculate $3 \cdot 2^5$.

Is it $6^5 = 1,296$ or $3 \cdot 32 = 96$?

We could use parentheses to be absolutely clear about how we want the expression to be evaluated:

$$(3 \cdot 2)^5 \text{ or } 3 \cdot (2)^5.$$

However, an **order of operations** has been agreed upon that eliminates confusion when parentheses are not present.

The order of operations is as follows:

1. Simplify within parentheses.
2. Simplify any exponents.
3. Perform multiplication and division from left to right.
4. Perform addition and subtraction from left to right.

So, using the correct order of operations, the expression $3 \cdot 2^5$ equals

$$3 \cdot 32 = 96.$$

Example D

Simplify $5 + 7^2 - (3+9)$.

$$\begin{aligned} 5 + 7^2 - (3+9) &= 5 + 7^2 - 12 \\ &= 5 + 49 - 12 \\ &= 54 - 12 \\ &= 42 \end{aligned}$$

Extended Example 6a

Simplify $-6 + 2^3 \cdot (1+6) \div 14$.

Hint: Simplify within parentheses first.

Step 1:

$-6 + 2^3 \cdot (7) \div 14$

Hint: Simplify exponents next.

Step 2:

$= -6 + 8 \cdot 7 \div 14$

Hint: Follow the order of operations: multiplication first.

Step 3:

$= -6 + 56 \div 14$

Hint: Follow the order of operations: division next.

Answer:

$= -6 + 4 = -2$

Extended Example 6b

Simplify $5 - 3^2 \cdot (3-5) \div 18$.

Hint: Simplify within parentheses first.

Step 1:

$= 5 - 3^2 \cdot (-2) \div 18$

Hint: Simplify exponents next.

Step 2:

$= 5 - 9 \cdot (-2) \div 18$

Hint: Follow the order of operations: multiplication first.

Step 3:

$= 5 + 18 \div 18$

Hint: Follow the order of operations: division next.

Answer:

$= 5 + 1 = 6$

Extended Example 6c
Simplify $-6+5^4\cdot(3-4)\div 5$.

Hint: Simplify within parentheses first.
Step 1:
$=-6+5^4\cdot(-1)\div 5$

Hint: Simplify exponents next.
Step 2:
$=-6+625\cdot(-1)\div 5$

Hint: Follow the order of operations: multiplication first.
Step 3:
$=-6-625\div 5$
Hint: Follow the order of operations: division next.
Answer:
$=-6-125$

$=-131$

Consider the following examples. Some find it helpful to keep in mind that negatives are equivalent to multiplication by -1: $-A=(-1)\cdot A$.

Examples:

Simplify $(-2)^3$. $(-2)^3=(-2)(-2)(-2)=-8$

Simplify -2^3. $-2^3=(-1)\cdot 2^3=(-1)\cdot 8=-8$

Simplify $(-3)^2$. $(-3)^2=(-3)(-3)=9$

Simplify -3^2. $-3^2=(-1)\cdot 3^2=(-1)\cdot 9=-9$

Notes:

- A negative number raised to an even exponent is positive, but a negative number raised to an odd exponent is negative. Notice the use of the order of operations in the examples above: exponents must be simplified before multiplication is performed.
- The negative sign is not part of the base unless it is included in parentheses.

Example E
Write -16 as a power with integers as the base and exponent.

$-16=-4^2$; also, $-16=-2^4$.

However, $(-4)^2=16$ and $(-2)^4=16$.

Example F
Write -8 as a power with integers as the base and exponent.

$-8=-2^3$; also, $-8=(-2)^3$.

Example G

Does $(-5)^2 = -5^2$?

No. $(-5)^2 = 25$, but $-5^2 = -25$

Example H

Write 9 as a power of 3.

When you are asked to "write A as a power of B," ask yourself, "B to what power equals A?":

$$B^{\boxed{?}} = A$$

For this problem we ask, "3 to what power equals 9?" $3^{\boxed{?}} = 9 \quad \Rightarrow \quad 3^2 = 9$.

Question: Write $1,000$ as a power of 10.

Answer: In other words, 10 to what power equals $1,000$?

$$10^{\boxed{?}} = 1000 \quad \Rightarrow \quad 10^3 = 1000.$$

End of Lesson