Chapter 3 Section 3 Lesson
Additional Rules for Exponents

Introduction

In this lesson we'll examine some additional rules that govern the behavior of exponents. The rules should be memorized; they will be used often in the remaining chapters. These additional rules also follow from the definition of exponents given below.

$$x^n = \overbrace{x \cdot x \cdot x \cdots x}^{n \text{ of these}}.$$

The first new rule is the familiar cancellation rule for fractions, expressed in terms of exponents.

> **Rule 6:** $\dfrac{x^m}{x^n} = x^{m-n}$, for any real number x, $x \neq 0$, and any integers m and n.

Example:

$$\frac{2^7}{2^4} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{\cancel{2 \cdot 2 \cdot 2 \cdot 2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2 \cdot 2 \cdot 2 \cdot 2}} = 2 \cdot 2 \cdot 2 = 2^3 = 8$$

The four $2s$ in the denominator cancelled out four $2s$ in the numerator. This removed four of the seven $2s$, leaving three $2s$ in the numerator. That is the essence of this rule – cancellation. But using Rule 6 is simpler than canceling:

$$\frac{2^7}{2^4} = 2^{7-4} = 2^3 = 8$$

Viewing the animation in the course online might also help you understand this concept.

Example A below shows that without Rule 6 there would be a lot of canceling!

Example A

$$\frac{A^{555}}{A^{333}} = ?$$

$$\frac{A^{555}}{A^{333}} = A^{555-333} = A^{222}$$

It's clear from the examples on the previous screen that Rule 6 is true when the exponent in the numerator, m, is greater than the exponent in the denominator, n. Extending Rule 6 so that it applies for all integer values of m and n requires some new definitions.

Consider what happens when $m = n$. Clearly, $\frac{x^m}{x^m} = 1$.

But applying Rule 6, we get $\frac{x^m}{x^m} = x^{m-m} = x^0$.

This leads to a new rule. This new rule is: anything raised to the zero power equals 1.

Rule 7: $x^0 = 1$ for any real number $x \neq 0$.

Examples:

$$5^0 = 1, \quad 917^0 = 1, \quad (-2)^0 = 1, \quad \pi^0 = 1, \quad A^0 = 1, \quad \left(\frac{A^2 - 3}{\sqrt{A^2 + 1}}\right)^0 = 1$$

We also need to define negative exponents to make Rule 6 true when $m < n$.

Consider: $\frac{x^2}{x^3} = \frac{x \cdot x}{x \cdot x \cdot x} = \frac{\cancel{x} \cancel{x}}{\cancel{x} \cancel{x} \cdot x} = \frac{1}{x}$.

On the other hand, using Rule 6, we get: $\frac{x^2}{x^3} = x^{2-3} = x^{-1}$.

This leads us to the conclusion: $x^{-1} = \frac{1}{x}$. Using this definition, all the previous rules of exponents are true even when m and n are negative. The next rule is a generalization of what we just learned.

Rule 8 (Definition of Negative Exponent):
$x^{-n} = \frac{1}{x^n}$, for any real number x, $x \neq 0$, and for any integer n.

Examples: $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$.

$$\frac{2^4}{2^7} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{\cancel{2 \cdot 2 \cdot 2 \cdot 2}}{\cancel{2 \cdot 2 \cdot 2 \cdot 2} \cdot 2 \cdot 2 \cdot 2} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{2^3} = \frac{1}{8}$$

As you can see, four of the 2s in the denominator canceled out the four 2s in the numerator, leaving three 2s in the denominator. Applying Rule 6 is simpler:

$$\frac{2^4}{2^7} = 2^{4-7} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Viewing the animation in the course online might also help you understand this concept:

The table below contains all of the exponent rules (or "properties") we've seen so far. The Flashcard activity in the course online may help you memorize these properties. You may also want to print the table so you can refer to it for the remainder of this lesson and when working on the problems for this section.

Properties of Exponents	
Definition of Exponent: $a^n = \overbrace{a \cdot a \cdot a \cdots a}^{n \text{ of these}}$	
1	$a^1 = a$
2	$a^m \cdot a^n = a^{m+n}$
3	$(ab)^n = a^n \cdot b^n$
4	$(a^m)^n = a^{m \cdot n}$
5	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6	$\frac{a^m}{a^n} = a^{m-n}$
7	$a^0 = 1$
8	$a^{-n} = \frac{1}{a^n}$

Note: The variable a is used to state the properties of exponents in the table above and x is used in other parts of the lesson. Remember that the rules are the same no matter what variable is used.

Also, note that you do not need to remember the number of each rule; it's the rules that are important, not the order in which they are presented.

You know that multiplication is a shortcut for addition:

$$a \cdot n = \overbrace{a + a + \cdots + a}^{n \text{ of these}}.$$

In the same way, exponentiation is a shortcut for multiplication:

$$a^n = \overbrace{a \cdot a \cdot a \cdot \cdots \cdot a}^{n \text{ of these}}.$$

So, exponentiation distributes over multiplication just as multiplication distributes over addition.

- The distributive law of multiplication over addition: $(a + b) \cdot n = a \cdot n + b \cdot n$
- The distributive law of exponentiation over multiplication: $(ab)^n = a^n \cdot b^n$

You'll recognize the distributive law of exponentiation over multiplication as Rule 3 in the Properties of Exponents table above.

Example B

Rewrite $\left(8a^{-4}b^{-2}\right)^{-2}$ without using negative exponents.

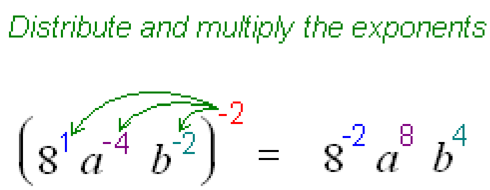
$$\left(8a^{-4}b^{-2}\right)^{-2} \overset{\boxed{1}}{=} \left(8^{1}a^{-4}b^{-2}\right)^{-2} \overset{\boxed{3}}{=} \left(8^{1}\right)^{-2}\left(a^{-4}\right)^{-2}\left(b^{-2}\right)^{-2}$$

$$\overset{\boxed{4}}{=} 8^{1\cdot(-2)}a^{-4\cdot(-2)}b^{-2\cdot(-2)} = 8^{-2}a^{8}b^{4}$$

$$\overset{\boxed{8}}{=} \frac{1}{8^{2}}\cdot a^{8}b^{4} = \frac{1}{64}\cdot\frac{a^{8}b^{4}}{1} = \frac{a^{8}b^{4}}{64}$$

The numbers in the boxes above refer to the rules for exponents in the table on page 3.

There is a shorter way to perform the calculations in Example B by combining steps 3 and 4 together into one step. Use the distributive law of exponentiation over multiplication to distribute the exponent to each factor. This is shown in the animation in the course online.



This image represents an animation that can only be seen in the course online.

The exponent outside the parentheses, -2, distributes to each factor inside the parentheses:

-2 times 1 equals -2
-2 times -4 equals 8
-2 times -2 equals 4.

Notice how much less work it would have been to use this shortcut for Example B. Next, you'll be guided step by step through simplifying problems similar to Example B, using this shortcut.

Question: Simplify $\left(5^{-2}\cdot 2^{-3}\right)^{-2}$.

Answer: Multiply all the exponents by -2: $\left(5^{-2}\cdot 2^{-3}\right)^{-2} = 5^{4}\cdot 2^{6}$

Then multiply: $= 625\cdot 64 = 40{,}000$

Extended Example 1a

Simplify $\left(2^{-4} \cdot 3^{2}\right)^{-2}$, expressing it without negative exponents.

Hint: Multiply all the exponents by -2.

Step 1:

$= 2^{8} \cdot 3^{-4}$

Hint: Use Rule 8, $a^{-n} = \dfrac{1}{a^{n}}$.

Step 2:

$= 2^{8} \cdot \dfrac{1}{3^{4}}$

Hint: Write this as a single fraction.

Answer:

$= \dfrac{2^{8}}{1} \cdot \dfrac{1}{3^{4}}$

$= \dfrac{2^{8} \cdot 1}{1 \cdot 3^{4}}$

$= \dfrac{2^{8}}{3^{4}}$

$= \dfrac{256}{81}$

Extended Example 1b

Simplify $\left(3^{2} \cdot 2^{-3}\right)^{-3}$.

Hint: Multiply all the exponents by the distributing the -3.

Step 1:

$= 3^{-6} \cdot 2^{9}$

Hint: Use Rule 8, $a^{-n} = \dfrac{1}{a^{n}}$.

Step 2:

$= \dfrac{1}{3^{6}} \cdot 2^{9}$

Hint: Write this as a single fraction.

Answer:

$= \dfrac{1}{3^{6}} \cdot \dfrac{2^{9}}{1}$

$= \dfrac{1 \cdot 2^{9}}{3^{6} \cdot 1}$

$= \dfrac{2^{9}}{3^{6}} = \dfrac{512}{729}$

Extended Example 2a

Simplify $\left(2^{-2}\, x^{7} y^{-5}\right)^{-3}$, expressing it without negative exponents.

Hint: Multiply all the exponents by -3.

Step 1:

$= 2^{6}\, x^{-21} y^{15}$

Hint: Use Rule 8, $a^{-n} = \dfrac{1}{a^{n}}$, and simplify 2^{6}.

Step 2:

$= 64 \cdot \dfrac{1}{x^{21}} \cdot y^{15}$

Hint: Multiply.

Answer:

$= \dfrac{64}{1} \cdot \dfrac{1}{x^{21}} \cdot \dfrac{y^{15}}{1}$

$= \dfrac{64 \cdot 1 \cdot y^{15}}{1 \cdot x^{21} \cdot 1}$

$= \dfrac{64 y^{15}}{x^{21}}$

Extended Example 2b

Simplify $\left(3^{-2}\, u^{-8} v^{13}\right)^{-2}$, expressing it without negative exponents.

Hint: Multiply all the exponents by the distributing the -2.

Step 1:

$= 3^{4}\, u^{16} v^{-26}$

Hint: Use Rule 8, $a^{-n} = \dfrac{1}{a^{n}}$, and simplify 3^{4}.

Step 2:

$= 81 \cdot u^{16} \cdot \dfrac{1}{v^{26}}$

Hint: Write this as a single fraction.

Answer:

$= \dfrac{81}{1} \cdot \dfrac{u^{16}}{1} \cdot \dfrac{1}{v^{26}}$

$= \dfrac{81 \cdot u^{16} \cdot 1}{1 \cdot 1 \cdot v^{26}}$

$= \dfrac{81 u^{16}}{v^{26}}$

Extended Example 2c

Simplify $\left(3^2 u^{-7} v^{-9}\right)^{-5}$, expressing it without negative exponents.

Hint: Multiply all the exponents by -5.

Step 1:

$= 3^{-10} u^{35} v^{45}$

Hint: Use Rule 8, $a^{-n} = \dfrac{1}{a^n}$.

Step 2:

$= \dfrac{1}{3^{10}} \cdot u^{35} v^{45}$

Hint: Multiply.

Answer:

$= \dfrac{1}{3^{10}} \cdot u^{35} v^{45}$

$= \dfrac{1}{3^{10}} \cdot \dfrac{u^{35} v^{45}}{1}$

$= \dfrac{1 \cdot u^{35} v^{45}}{3^{10} \cdot 1}$

$= \dfrac{u^{35} v^{45}}{3^{10}}$ or $\dfrac{u^{35} v^{45}}{59{,}049}$

Extended Example 3a

Simplify $\left(4a^{-7}\right)^{-2}\left(4^{-2} a^9\right)^{-3}$, expressing it without negative exponents.

Hint: Use Rule 1, $a^1 = a$. We need the 4 to have an exponent of 1.

Step 1:

$= \left(4^1 a^{-7}\right)^{-2}\left(4^{-2} a^9\right)^{-3}$

Hint: Multiply all the exponents in the first set of parentheses by -2 and all the exponents in the second set by -3.

Step 2:

$= 4^{-2} a^{14} 4^6 a^{-27}$

Hint: Group the numerical factors on the left.

Step 3:

$= 4^{-2} 4^6 a^{14} a^{-27}$

Hint: Use Rule 2, $a^m \cdot a^n = a^{m+n}$, on the 4s and on the $a\text{s}$.

Step 4:

$= 4^{-2} 4^6 a^{14} a^{-27}$

$= 4^{(-2)+6} a^{14+(-27)}$

$= 4^4 a^{-13}$

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Hint: Use Rule 8, $a^{-n} = \dfrac{1}{a^n}$.

Step 5:

$$= 4^4 \cdot \frac{1}{a^{13}}$$

Hint: Multiply.

Answer:

$$= \frac{4^4}{1} \cdot \frac{1}{a^{13}} = \frac{4^4 \cdot 1}{1 \cdot a^{13}} = \frac{4^4}{a^{13}} = \frac{256}{a^{13}}$$

Extended Example 3b

Simplify $\left(5^{-1}x^{-12}\right)^{-3}\left(5^2x\right)^{-2}$, expressing it without negative exponents.

Hint: Use Rule 1, $a^1 = a$. We need x to have an exponent of 1.

Step 1:

$$= \left(5^{-1}x^{-12}\right)^{-3}\left(5^2x^1\right)^{-2}$$

Hint: Multiply all the exponents in the first set of parentheses by -3 and all the exponents in the second set by -2.

Step 2:

$$= 5^3x^{36} \cdot 5^{-4}x^{-2}$$

Hint: Write the number factors on the left.

Step 3:

$$= 5^3 \cdot 5^{-4}x^{36}x^{-2}$$

Hint: Use Rule 2, $a^m \cdot a^n = a^{m+n}$, on the 5s and on the xs.

Step 4:

$$= 5^3 \cdot 5^{-4}x^{36}x^{-2}$$

$$= 5^{3+(-4)}x^{36+(-2)}$$

$$= 5^{-1}x^{34}$$

Hint: Use Rule 8, $a^{-n} = \dfrac{1}{a^n}$.

Step 5:

$$= \frac{1}{5^1} \cdot x^{34} = \frac{1}{5} \cdot x^{34}$$

Hint: Write this as a single fraction.

Answer:

$$= \frac{1}{5} \cdot \frac{x^{34}}{1} = \frac{1 \cdot x^{34}}{5 \cdot 1} = \frac{x^{34}}{5}$$

Extended Example 3c

Simplify $\left(2^3 y^{-6}\right)^{-5}\left(2^{-5} y^3\right)^{-3}$, expressing it without negative exponents.

Hint: Multiply all the exponents in the first set of parentheses by -5 and all the exponents in the second set by -3.

Step 1:

$= 2^{-15} y^{30} \cdot 2^{15} y^{-9}$

Hint: Write the numeric factors on the left.

Step 2:

$= 2^{-15} \cdot 2^{15} y^{30} y^{-9}$

Hint: Use Rule 2, $a^m \cdot a^n = a^{m+n}$, on the 2s and on the ys.

Step 3:

$= 2^{-15} \cdot 2^{15} y^{30} y^{-9}$

$= 2^{-15+15} y^{30+(-9)}$

$= 2^0 y^{21}$

Hint: Use Rule 7, $a^0 = 1$

Answer:

$= 1 \cdot y^{21} = y^{21}$

<u>The Elevator Rule</u>

One shortcut that you might find useful is something we can call "the **elevator rule**" (this is not a common term; it was invented by this author to help you remember the shortcut). This shortcut comes from Rule 8 combined with the rules for manipulating fractions.

Consider the expression $\dfrac{a^{-4}b^{-3}}{c^5 d^{-6}}$. Notice that the numerator and the denominator are both completely factored as products. This is the only situation where this shortcut works; **if there is any addition or subtraction in the fraction, do NOT use this technique.**

To help you remember the elevator rule, think of the sign of the exponent as describing the "state of mind" of its corresponding variable. For example, variable a above has a negative state of mind and is unhappy. Why is a unhappy, you ask? Because she wants to be downstairs in the denominator but she's stuck up in the numerator. Similarly, b is unhappy about being upstairs, and d is unhappy about being downstairs. This is where the elevator comes in—they can each take the elevator to go wherever makes them happy. Notice that c can stay where she is because she's already happy. After they all take their elevator rides, the expression is transformed into one where everybody is happy (where there are no negative exponents):

$$\frac{a^{-4}b^{-3}}{c^5 d^{-6}} = \frac{d^6}{a^4 b^3 c^5}$$

The negative exponents were eliminated in just one step. The animation in the course online shows the elevator rule in action.

Question: Simplify $\dfrac{2^{-3}x^{-1}y^{-2}}{3^{-2}z^{-8}}$, expressing it without negative exponents.

Answer: Use the elevator rule and then multiply the coefficients:

$$\frac{2^{-3}x^{-1}y^{-2}}{3^{-2}z^{-8}} = \frac{3^2z^8}{2^3x^1y^2} = \frac{9z^8}{8xy^2}$$

Question: Simplify $\dfrac{4^{-2}a^{-6}}{5^{-3}b^{-3}c^{-5}}$, expressing it without negative exponents.

Answer: Use the elevator rule and then multiply the coefficients:

$$\frac{4^{-2}a^{-6}}{5^{-3}b^{-3}c^{-5}} = \frac{5^3b^3c^5}{4^2a^6} = \frac{125b^3c^5}{16a^6}$$

Question: Simplify $\dfrac{7^{-2}u^{-45}v^{-28}}{3^{-4}w^{-41}z^{-62}}$, expressing it without negative exponents.

Answer: Use the elevator rule and then multiply the coefficients:

$$\frac{7^{-2}u^{-45}v^{-28}}{3^{-4}w^{-41}z^{-62}} = \frac{3^4w^{41}z^{62}}{7^2u^{45}v^{28}} = \frac{81w^{41}z^{62}}{49u^{45}v^{28}}$$

Extended Example 4a

Simplify $\left(\dfrac{5^{-2}u^3}{2^2z^{-6}v^{-4}}\right)^{-2}$, expressing it without negative exponents.

Hint: Use Rule 5, $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$.

Step 1:

$$= \frac{\left(5^{-2}u^3\right)^{-2}}{\left(2^2z^{-6}v^{-4}\right)^{-2}}$$

Hint: Use Rule 3, $(ab)^n = a^n \cdot b^n$.

Step 2:

$$= \frac{5^4u^{-6}}{2^{-4}z^{12}v^8}$$

Hint: Use the elevator rule.

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Step 3:

$$= \frac{2^4 \cdot 5^4}{z^{12} v^8 u^6}$$

Hint: Multiply.

Answer:

$$= \frac{16 \cdot 625}{z^{12} v^8 u^6} = \frac{10,000}{u^6 v^8 z^{12}}$$

Note that the final solution is expressed with the variables in alphabetical order, which is standard form.

Extended Example 4b

Simplify $\left(\frac{6A^{-3}}{5^{-1}B^{-6}}\right)^{-2}$, expressing it without negative exponents.

Hint: Use Rule 5, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

Step 1:

$$= \frac{\left(6A^{-3}\right)^{-2}}{\left(5^{-1}B^{-6}\right)^{-2}}$$

Hint: Use Rule 1, $a^1 = a$. (We need the 6 to have an exponent of 1.)

Step 2:

$$= \frac{\left(6^1 A^{-3}\right)^{-2}}{\left(5^{-1}B^{-6}\right)^{-2}}$$

Hint: Use Rule 3, $(ab)^n = a^n \cdot b^n$.

Step 3:

$$= \frac{6^{-2} A^6}{5^2 B^{12}}$$

Hint: Use the elevator rule.

Step 4:

$$= \frac{A^6}{6^2 \cdot 5^2 B^{12}}$$

Hint: Multiply.

Answer:

$$= \frac{A^6}{36 \cdot 25 B^{12}} = \frac{A^6}{900 B^{12}}$$

Extended Example 4c

Simplify $\left(\dfrac{7^{-1}p^{-7}}{2^{-3}q^6r^4}\right)^{-3}$, expressing it without negative exponents.

Hint: Use Rule 5, $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$.

Step 1:

$$=\frac{\left(7^{-1}p^{-7}\right)^{-3}}{\left(2^{-3}q^6r^4\right)^{-3}}$$

Hint: Use Rule 3, $(ab)^n = a^n \cdot b^n$.

Step 2:

$$=\frac{7^3p^{21}}{2^9q^{-18}r^{-12}}$$

Hint: Use the elevator rule.

Step 3:

$$=\frac{7^3p^{21}q^{18}r^{12}}{2^9}$$

Hint: Multiply.

Answer:

$$=\frac{343p^{21}q^{18}r^{12}}{512}$$

Extended Example 5a

Simplify $\left(\dfrac{2^{-3}x^{-3}}{2^{-5}x^{-2}}\right)^{-3}$, expressing it without negative exponents.

Hint: Simplify the expression in the parentheses first, using Rule 6, $\dfrac{a^m}{a^n} = a^{m-n}$.

Step 1:

$$=\left(\frac{2^{-3}x^{-3}}{2^{-5}x^{-2}}\right)^{-3} = \left(\frac{2^{-3}}{2^{-5}}\cdot\frac{x^{-3}}{x^{-2}}\right)^{-3}$$

$$=\left(2^{-3-(-5)}\cdot x^{-3-(-2)}\right)^{-3}$$

$$=\left(2^{-3+5}\cdot x^{-3+2}\right)^{-3}$$

$$=\left(2^2x^{-1}\right)^{-3}$$

Hint: Distribute the -3 using Rule 3, $(ab)^n = a^n \cdot b^n$, and Rule 4, $\left(a^m\right)^n = a^{m\cdot n}$.

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Step 2:
$= 2^{-6}x^{3}$

Hint: Use Rule 8, $a^{-n} = \dfrac{1}{a^{n}}$, or the elevator rule to eliminate the negative exponent.

Step 3:
$= \dfrac{1}{2^{6}} \cdot \dfrac{x^{3}}{1} = \dfrac{x^{3}}{2^{6}}$

Hint: Simplify the denominator.

Answer:
$= \dfrac{x^{3}}{64}$

Extended Example 5b

Simplify $\left(\dfrac{3^{-7}y^{-8}}{3^{-6}y^{-9}}\right)^{-3}$, expressing it without negative exponents.

Hint: Simplify the expression in the parentheses first, using Rule 6, $\dfrac{a^{m}}{a^{n}} = a^{m-n}$.

Step 1:

$$= \left(\frac{3^{-7}y^{-8}}{3^{-6}y^{-9}}\right)^{-3} = \left(\frac{3^{-7}}{3^{-6}} \cdot \frac{y^{-8}}{y^{-9}}\right)^{-3}$$

$$= \left(3^{-7-(-6)} \cdot y^{-8-(-9)}\right)^{-3}$$

$$= \left(3^{-7+6} \cdot y^{-8+9}\right)^{-3}$$

$$= \left(3^{-1}y^{1}\right)^{-3}$$

Hint: Distribute the -3 using Rule 3, $(ab)^{n} = a^{n} \cdot b^{n}$, and Rule 4, $\left(a^{m}\right)^{n} = a^{m \cdot n}$.

Step 2:
$= 3^{3}y^{-3}$

Hint: Use Rule 8, $a^{-n} = \dfrac{1}{a^{n}}$, or the elevator rule to eliminate the negative exponent.

Step 3:
$= 3^{3} \cdot \dfrac{1}{y^{3}} = \dfrac{3^{3}}{1} \cdot \dfrac{1}{y^{3}} = \dfrac{3^{3}}{y^{3}}$

Hint: Simplify the numerator.

Answer:
$= \dfrac{27}{y^{3}}$

Extended Example 5c

Simplify $\left(\dfrac{2^{-3}s^5t^{-8}}{2^2s^{-9}t^7}\right)^{-2}$, expressing it without negative exponents.

Hint: Simplify the expression in the parentheses first, using Rule 6, $\dfrac{a^m}{a^n}=a^{m-n}$.

Step 1:

$$=\left(\frac{2^{-3}s^5t^{-8}}{2^2s^{-9}t^7}\right)^{-2}=\left(\frac{2^{-3}}{2^2}\cdot\frac{s^5}{s^{-9}}\cdot\frac{t^{-8}}{t^7}\right)^{-2}$$

$$=\left(2^{-3-2}\cdot s^{5-(-9)}\cdot t^{-8-7}\right)^{-2}$$

$$=\left(2^{-5}\cdot s^{5+9}\cdot t^{-15}\right)^{-2}$$

$$=\left(2^{-5}\cdot s^{14}\cdot t^{-15}\right)^{-2}$$

Hint: Distribute the -2 using Rule 3, $(ab)^n=a^n\cdot b^n$, and Rule 4, $\left(a^m\right)^n=a^{m\cdot n}$.

Step 2:

$$=2^{10}s^{-28}t^{30}$$

Hint: Use Rule 8, $a^{-n}=\dfrac{1}{a^n}$, or the elevator rule to eliminate the negative exponent.

Step 3:

$$=2^{10}\cdot\frac{1}{s^{28}}\cdot t^{30}$$

$$=\frac{2^{10}}{1}\cdot\frac{1}{s^{28}}\cdot\frac{t^{30}}{1}$$

$$=\frac{2^{10}t^{30}}{s^{28}}$$

Hint: Simplify the coefficient in the numerator.

Answer:

$$=\frac{1024t^{30}}{s^{28}}$$

End of Lesson