

Chapter 3 Section 4 Lesson

From Words to Algebraic Expressions

Introduction

In this lesson we'll describe the process of translating certain English phrases and sentences into algebraic expressions and equations. Usually, real-world math problems are expressed first in words. Translating the words into equations allows us to apply the rules of algebra to solve the problems. This lesson introduces the process of applying mathematics to the real world. In fact, math can be found almost everywhere—some physicists have discovered that the universe itself seems to be written in the language of mathematics.

Example A

Write an algebraic expression for the “sum of x and 9.”

The word “sum” indicates addition. So, the algebraic expression is $x + 9$. Note that $9 + x$ is also correct.

“Sum of x and 9”

$$x + 9$$

This image represents an animation that can only be seen in the course online.

Example B

Write an algebraic expression for “13 decreased by y .”

The phrase “decreased by” indicates subtraction. The algebraic expression for “13 decreased by y ” is $13 - y$. Note that $y - 13$ is not correct—the order in which you subtract is important. Other word expressions for $13 - y$ are “13 minus y ,” “ y less than 13,” and “ y subtracted from 13.”

“13 decreased by y ”

$$13 - y$$

This image represents an animation that can only be seen in the course online.

Example C

Write an algebraic expression for “the sum of a number and 12.”

The phrase “a number” indicates an unknown quantity. We can use any variable to represent it in our algebraic expression. For example, $b + 12$.

Example D

Write an algebraic expression for “the product of s^3 and 5.”

The word “product” indicates multiplication, so: $5s^3$, which is a monomial.

Example E

Write an expression for “9 more than the product of x and $2y$.”

In this case, you add 9 to the “product of x and $2y$.” So, the expression is $x \cdot 2y + 9$. The standard form for this expression is $2xy + 9$.

Example F

Write an algebraic expression for “one-eighth of n .”

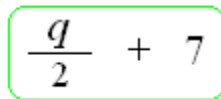
A “fraction of” or “percent of” a number or variable indicates that you multiply the number by the fraction or percent. An algebraic expression for “one-eighth of n ” is $\frac{1}{8}n$, which can also be written as $\frac{n}{8}$ or $n \div 8$. Notice that these are monomials whose coefficients are all $\frac{1}{8}$.

Example G

Write an algebraic expression for “seven added to one-half of a number.”

An algebraic expression is $\frac{1}{2}q + 7$ or $\frac{q}{2} + 7$.

“seven added to one-half of a number”


$$\frac{q}{2} + 7$$

This image represents an animation that can only be seen in the course online.

Example H

Write an expression for “9 times the sum of x and $2y$.”

You will need parentheses to indicate that the entire “sum of x and $2y$ ” is multiplied by 9. The algebraic expression is $9(x + 2y)$.

Example I

Write an expression for “one-fifth of xy decreased by z .”

Translating the phrase directly, we get $\frac{1}{5}xy - z$ which simplifies to $\frac{xy}{5} - z$.

Example J

Write the algebraic expression for “ Q minus 3, divided by 6.”

The comma indicates a pause, showing that the difference is grouped together. So, the algebraic expression reflects the grouping: $\frac{Q - 3}{6}$.

Example K

Write the algebraic expression for “4 times R plus 6.”

Since there is no punctuation to indicate grouping in the phrase, we translate directly and get $4R + 6$.

Example L

Write the algebraic expression for the phrase “ ax^2 plus b divided by c .”

Again, since there is no grouping, this means $ax^2 + b \div c$ or $ax^2 + \frac{b}{c}$.

We often want to say that one expression is related to another expression. Relating one expression to another results in what are called **algebraic equations** or **algebraic inequalities**.

These are the most common word expressions that indicate such relationships:

is equal to	=
is not equal to	≠
is less than	<
is less than or equal to	≤
is greater than	>
is greater than or equal to	≥

These relational symbols are the verbs used to turn phrases into sentences. Phrases never contain these symbols. Complete sentences always contain one of these symbols.

Example M

Translate “ X less than Y ” into an algebraic expression.

Notice that this is a phrase without a verb, not a sentence, so no inequality symbol or equal sign is used.

Often it can be helpful to substitute numbers in place of variables. In this case, think “2 less than 5.” Then determine “what is two less than five?” “3 is 2 less than 5.” So, how do you get 3 using the phrase “2 less than 5?” You subtract: $5 - 2 = 3$. In other words: “2 less than 5” translates as $5 - 2$. So, “ X less than Y ” translates as $Y - X$.

Example N

Translate “ X is less than Y ” into an algebraic expression.

This is a phrase with a verb (“is”), which makes it a sentence; in this case an inequality symbol is used. To see which direction the inequality points, remember: the alligator, $<$, always eats the bigger number. In this example Y is bigger, so the sentence “ X is less than Y ” translates as $X < Y$.

Note that the phrase “less than” means subtraction, but the phrase “is less than” means an inequality—the verb “is” indicates that a relational symbol should be used.

Example O

Write an algebraic equation for “8 more than x is equal to 17.”

The algebraic equation is $x + 8 = 17$.

Example P

What is an algebraic equation for “7 less than the product of 2 and n is equal to the product of 3 and n ”?

It helps to insert parentheses into the sentence, to mirror the grammar:

7 less than (the product of 2 and n) = (the product of 3 and n).

Now start translating the sub-expressions in the parentheses into algebraic expressions:

7 less than $(2n) = (3n)$. So: $2n - 7 = 3n$.

Example Q

Write an algebraic inequality for “one-fifth of x is greater than x minus 4.”

The inequality is $\frac{1}{5}x > x - 4$.

Extended Example 1a

Write an algebraic inequality for “5 added to the quotient of a number and 3 is less than or equal to 7 times the sum of the number and 2.” For the number, use the variable n .

Hint: Insert parentheses around the sub-phrases in the sentence, with the appropriate inequality symbol.

Step 1:

“5 added to (the quotient of a number and 3) \leq 7 times (the sum of the number and 2).”

Hint: Replace references to the number with the variable n .

Step 2:

“5 added to (the quotient of n and 3) \leq 7 times (the sum of n and 2)”

Hint: Translate the phrases in parentheses into algebraic expressions.

Step 3:

“5 added to $\frac{n}{3} \leq$ 7 times $(n + 2)$ ”

Hint: Finish the translation.

Answer:

$$5 + \frac{n}{3} \leq 7(n + 2)$$

Extended Example 1b

Write an algebraic inequality for the sentence: “7 subtracted from the product of a number and 5 is greater than or equal to 2 times the difference of that number and 9.” For the number, use the variable n .

Hint: Insert parentheses around the sub-phrases in the sentence, with the appropriate inequality symbol.

Step 1:

“7 subtracted from (the product of a number and 5) \geq 2 times (the difference of a number and 9)”

Hint: Replace references to the number with the variable n .

Step 2:

“7 subtracted from (the product of n and 5) \geq 2 times (the difference of n and 9)”

Hint: Translate the phrases in parentheses into algebraic expressions.

Step 3:

“7 subtracted from $(5n) \geq$ 2 times $(n - 9)$ ”

Hint: Finish the translation. Drop unnecessary parentheses.

Answer:

$$5n - 7 \geq 2(n - 9)$$

Extended Example 1c

Write an algebraic inequality for the sentence: "4 more than the product of a number and 10 is less than 6 less than the quotient of that number and 2." For the number, use the variable w .

Hint: Insert parentheses around the sub-phrases in the sentence, with the appropriate inequality symbol. Also replace references to the number with w .

Step 1:

"4 more than (the product of w and 10) $<$ 6 less than (the quotient of w and 2)"

Hint: Translate the phrases in parentheses into algebraic expressions.

Step 2:

"4 more than $(10w)$ $<$ 6 less than $\left(\frac{w}{2}\right)$ "

Hint: Finish the translation. Drop unnecessary parentheses.

Answer:

$$10w + 4 < \frac{w}{2} - 6$$

A Word to the Wise

Notice in this example the difference between the phrase "less than" and the phrase "is less than." Without that important little verb, "is," the phrase implies subtraction not the relational symbol, $<$.

Extended Example 2a

Write an algebraic equation for "4 less than the product of a number and 5 is 2 less than the sum of twice the number and 1." For the number, use variable x .

Hint: Insert parentheses around the sub-phrases in the sentence and insert the appropriate relational symbol.

Step 1:

"4 less than (the product of a number and 5) $=$ 2 less than (the sum of twice the number and 1)"

Hint: Translate the phrases in parentheses into algebraic expressions.

Step 2:

"4 less than $(5x)$ $=$ 2 less than $(2x + 1)$ "

Hint: Finish the translation.

Step 3:

$$(5x) - 4 = (2x + 1) - 2$$

Hint: Remove the parentheses.

Answer:

$$5x - 4 = 2x + 1 - 2, \text{ which could be simplified to } 5x - 4 = 2x - 1.$$

Extended Example 2b

Write an algebraic equation for the sentence: “8 more than the difference of a number and 2 equals 6 times the sum of that number and 1.” For the number, use x .

Hint: Insert parentheses around the sub-phrases in the sentence and insert the appropriate relational symbol.

Step 1:

“8 more than (the difference of x and 2) = 6 times (the sum of x and 1)”

Hint: Translate the phrases in the parentheses into algebraic expressions.

Step 2:

“8 more than $(x - 2)$ = 6 times $(x + 1)$ ”

Hint: Finish the translation. Drop unnecessary parentheses.

Answer:

$$(x - 2) + 8 = 6(x + 1)$$

$$x - 2 + 8 = 6(x + 1) \quad \text{or} \quad x + 6 = 6(x + 1)$$

Extended Example 2c

Write an algebraic equation for the sentence: “10 less than the sum of a number and 7 is 5 less than the quotient of that number and 8.” For the number, use x .

Hint: Insert parentheses around the sub-phrases in the sentence and insert the appropriate relational symbol.

Step 1:

“10 less than (the sum of x and 7) = 5 less than (the quotient of x and 8)”

Hint: Translate the phrases in the parentheses into algebraic expressions.

Step 2:

“10 less than $(x + 7)$ = 5 less than $\left(\frac{x}{8}\right)$ ”

Hint: Finish the translation. Drop unnecessary parentheses.

Answer:

$$(x + 7) - 10 = \left(\frac{x}{8}\right) - 5$$

$$x + 7 - 10 = \frac{x}{8} - 5 \quad \text{or} \quad x - 3 = \frac{x}{8} - 5$$

The tables below show some sample word phrases and how to write them algebraically.

$\left. \begin{array}{l} x \text{ plus } 8 \\ 8 \text{ added to } x \\ x \text{ increased by } 8 \\ 8 \text{ more than } x \\ \text{the sum of } x \text{ and } 8 \\ \text{the total of } x \text{ and } 8 \\ x \text{ and } 8 \text{ together} \end{array} \right\} x + 8$	$\left. \begin{array}{l} 8 \text{ times } x \\ \text{the product of } 8 \text{ and } x \\ x \text{ multiplied by } 8 \\ 8 \text{ multiplied by } x \\ 8 \text{ } x\text{s} \end{array} \right\} 8x$
$\left. \begin{array}{l} x \text{ minus } 8 \\ 8 \text{ subtracted from } x \\ x \text{ decreased by } 8 \\ 8 \text{ less than } x \\ 8 \text{ fewer than } x \\ \text{the difference of } x \text{ and } 8 \end{array} \right\} x - 8$	$\left. \begin{array}{l} x \text{ divided by } 8 \\ \text{one-eighth of } x \\ \text{an eighth part of } x \\ \text{the quotient of } x \text{ and } 8 \\ \text{the ratio of } x \text{ and } 8 \end{array} \right\} \frac{x}{8} \text{ or } x \div 8 \text{ or } \frac{1}{8}x$

$\left. \begin{array}{l} \text{is less than } 8 \\ \text{is smaller than } 8 \end{array} \right\} < 8$	$\left. \begin{array}{l} \text{equals } 8 \\ \text{is } 8 \\ \text{is equal to } 8 \end{array} \right\} = 8$
$\left. \begin{array}{l} \text{is greater than } 8 \\ \text{is more than } 8 \end{array} \right\} > 8$	
$\left. \begin{array}{l} \text{is greater than or equal to } 8 \\ \text{is less than or equal to } 8 \end{array} \right\} \geq 8$	$\left. \begin{array}{l} \text{does not equal } 8 \\ \text{is not } 8 \\ \text{is not equal to } 8 \end{array} \right\} \neq 8$
$\left. \begin{array}{l} \text{is greater than or equal to } 8 \\ \text{is less than or equal to } 8 \end{array} \right\} \leq 8$	

End of Lesson