**Chapter 3 Section 6 Lesson**
**Polynomials**

## Introduction

This lesson introduces polynomials and like terms.

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As we learned earlier, a **monomial** is a constant, a variable, or the product of constants and variables. Some examples of monomials follow:

$$3, \quad -7x, \quad 2xy, \quad -\frac{2}{3}x^3, \quad 14x^6y^2z^9$$

A **polynomial** is a monomial or the sum or difference of any number of monomials. The monomials listed above are examples of polynomials, as are the following:

$$1$$

$$-9x$$

$$2x+5$$

$$3x^2-7x+1$$

$$-5x^2y+y^3z^9-2x^3z^2$$

Some examples of expressions that are not polynomials follow.

| Expression | Why isn't it a polynomial? |
|---|---|
| $\frac{3x-1}{2x^4+5}$ | A polynomial cannot contain a variable in a denominator. |
| $4x^2+3\sqrt{x}$ | A polynomial cannot contain a variable under a radical sign. |
| $6x^2+\frac{2}{x}-3$ | There is a variable in a denominator. |

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**Question:** Is $7t^5-2t+1$ a polynomial?

*Answer:* Yes. It is the sum of the monomials $7t^5$, $-2t$, and $1$.

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**Question:** Is $\frac{2}{3}t+4-st+9x^2$ a polynomial?

*Answer:* Yes. It is the sum of the monomials $\frac{2}{3}t$, $4$, $-st$, and $9x^2$.

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**Question:** Is $\dfrac{x^2}{4y+1} - 3xy$ a polynomial?

*Answer:* No. If a fraction has a variable in its denominator, it is not a monomial. So, $\dfrac{x^2}{4y+1}$ is not a monomial and the given expression is not a polynomial.

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When a polynomial is written in its usual way, as a sum of monomials, then each monomial is a **term** of the polynomial. Monomials are terms, but so are many algebraic expressions that are not monomials. In general, a **term** in an algebraic expression is a monomial or any expression that is a product of factors.

If two or more terms in a polynomial have the same variables raised to the same power they are called **like terms**.

Example: In the polynomial $5x^2y - 9xy + 7x^2y - 4$, the first and third terms, $5x^2y$ and $7x^2y$, are like terms.

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*Notes:*

- Like terms can be added. This is called **combining like terms**. When you combine like terms, you add the coefficients of each term and leave the variable parts unchanged. Thus,
$$5x^2y - 9xy + 7x^2y - 4 = 12x^2y - 9xy - 4.$$

- The distributive property, $a \cdot c + b \cdot c = (a+b) \cdot c$, justifies combining like terms. Consider the like terms that were combined above. Using the distributive property:
$$5x^2y + 7x^2y = (5+7)x^2y = 12x^2y.$$

- Polynomials with two terms are called **binomials**. Polynomials with three terms are called **trinomials**.

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**Example A**

Simplify $8x^2y - 2xy + 3x^2y - 7$ by combining like terms.

$$8x^2y - 2xy + 3x^2y - 7 = 11x^2y - 2xy - 7$$

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**Example B**
Combine the like terms in the polynomials below.

a) $3 - 9r + 2r^3 + 4r - 6$

b) $4t^2 - 2 + t - 6t^2 - t$.

a) $3 - 9r + 2r^3 + 4r - 6 = 3 - 9r + 2r^3 + 4r - 6$

$= 2r^3 - 5r - 3$

b) $4t^2 - 2 + t - 6t^2 - t = -2t^2 - 2$ In this example, the $t$ terms add to $0$.

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**Question:** Combine like terms in the polynomial $3r + 2s + 5r - 7s$.

*Answer:* Identify the like terms:

$3r + 2s + 5r - 7s$

Then combine like terms:

$= 8r - 5s$

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We saw earlier that the degree of a monomial with only one variable is simply the degree of that variable. When a monomial has more than one variable, the degree of the monomial is the sum of the exponents of all the variables of the monomial.

Examples: The monomial $3x^2y^3$ has a degree of $5$, since $2 + 3 = 5$.

$5t^4$ has a degree of $4$.

$st = s^1t^1$ has a degree of $2$.

$-6xy^2 = -6x^1y^2$ has a degree of $3$.

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*Note:*

- Recall that constants such as $1$, $-5$, and $12.9$ are considered monomials. Such constant, but nonzero, monomials have degree zero. This makes sense in terms of the exponent rule, $x^0 = 1$. For example, using this peculiar way of writing the number one, we can show that the degree of the constant monomial known as the number $3$ is indeed zero:

  $3 = 3 \cdot 1 = 3 \cdot x^0 = 3x^0$. The degree of $3$ is zero, as is the degree of any constant.

- The only monomial whose degree is undefined is the monomial $0$.

The **degree** of a polynomial in standard form is the highest degree of any of its terms (assuming its like terms have all been combined).

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**Example C**

What is the degree of each monomial in the polynomial $4x - 2x^2y + 3xy^2 - 4$? What is the degree of the polynomial?

a) The degree of the first term, $4x = 4x^1$, is $1$.

b) The degree of the second term, $-2x^2y = -2x^2y^1$, is $3$.

c) The degree of the third term, $3xy^2 = 3x^1y^2$, is also $3$.

d) The degree of the fourth term, $-4 = -4 \cdot 1 = -4x^0$, is $0$.

e) Since $3$ is the highest degree of any term in the polynomial, the degree of this polynomial is $3$.

***Degree** of the polynomial = 3*

$$4x - 2x^2y + 3xy^2 - 4$$

*This image represents an animation that can only be seen in the course online.*

**Example D**

What is the degree of $3xy^3 - x^2y + 2x - 3xy^3 - 5 + 7y$?

The degree of this expression seems to be $4$ because the highest degree of the terms shown is $4$. *But wait…* The like terms have not been combined! When $3xy^3$ and $-3xy^3$ are combined, they cancel and the result is zero. So, the expression reduces to the polynomial $-x^2y + 2x - 5 + 7y$. Now, the highest degree of these terms is $3$. The polynomial actually has a degree of $3$.

**A Word to the Wise**

Always combine like terms before deciding the degree of a polynomial.

**Extended Example 1a**

Find the degree of the polynomial $2yz - xyz - 9y^4z^5 + 3yz + 9y^4z^5 + 2$.

Hint: Combine like terms.

*Step 1:*

$2yz - xyz - 9y^4z^5 + 3yz + 9y^4z^5 + 2 = 5yz - xyz + 2$

Hint: Find the degree of each term.

*Step 2:*

$5yz = 5y^1z^1$ has degree $1 + 1 = 2$;

$xyz = x^1y^1z^1$ has degree $1 + 1 + 1 = 3$;

$2 = 2x^0$ has degree $0 = 0$.

Hint: Select the greatest of those degrees.

*Answer:*

The polynomial has degree $3$.

*Note:* Once the like terms were combined, the polynomial above turned out to be a $3^{rd}$ degree trinomial (a polynomial with three terms).

**Extended Example 1b**

Find the degree of the polynomial $x^2yz^2 - y^3z^9 + 5x^4yz^2 + x^2yz + 2y^3z^9 - 5x^4yz^2 - y^3z^9$.

Hint: Don't panic! Many of the terms cancel once you combine like terms.

*Step 1:*

$$x^2yz^2 - y^3z^9 + 5x^4yz^2 + x^2yz + 2y^3z^9 - 5x^4yz^2 - y^3z^9$$
$$= x^2yz^2 + x^2yz$$

Hint: Find the degree of each term.

*Step 2:*

$x^2yz^2 = x^2y^1z^2$ has degree $2+1+2=5$;

$x^2yz = x^2y^1z^1$ has degree $2+1+1=4$;

Hint: Select the greatest of those degrees.

*Answer:*

The polynomial has degree $5$.

*Note:* Once the like terms were combined, the polynomial above turned out to be a $5^{th}$ degree binomial (a polynomial with two terms).

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**Example E**

Determine the degree of each polynomial below.

a) $8x^4 + 3x^3 + 6x^2 + 6x + 9$

b) $\frac{2}{3}n + 4 - mn + 9x^2$

c) $\frac{8s}{2t} + 9x - 5 - 9x$

d) $3 - 6x^2yz + 4xyz + 6yx^2z$

e) $9n - 6x^3 \cdot 0$

a) 4

b) 2

c) This is not a polynomial, since $\frac{8s}{2t}$ is not a monomial.

d) 3, since the second and fourth terms cancel out.

e) 1, because the second term equals zero and has no degree.

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We end this lesson by classifying the polynomials given as examples on page 1, noting the degree of each polynomial and whether it's a monomial, binomial, or trinomial:

$1$ . . . . . degree 0 monomial

$-9x$ . . . . . degree 1 monomial

$2x+5$ . . . . . degree 1 binomial

$3x^2 - 7x + 1$ . . . . . degree 2 trinomial

$-5x^2y + y^3z^9 - 2x^3z^2$ . . degree 12 trinomial

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**End of Lesson**