

Chapter 2 Section 5: Linear Inequalities

Introduction

Now we'll see what happens in the coordinate plane when we replace the equal sign in a linear equation with an inequality symbol. A line with equation

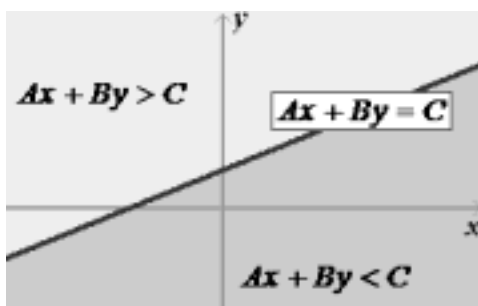
$$Ax + By = C$$

divides the coordinate plane into two regions. On one side of the line it's true for all points (x, y) that

$$Ax + By > C$$

while on the other side of the line, it's true that

$$Ax + By < C.$$



This is the case with any linear equation written in any form (for example, slope-intercept form, point-slope form, etc.), when the equal sign in the linear equation is replaced with an inequality symbol.

Example A

Graph the solution set of the inequality $2x + 3y \leq 6$.

Solving an inequality is a two-step process.

Step 1—Replace the inequality symbol with an equal sign and graph that line.

In this case, graph $2x + 3y = 6$. To do this, we need two points that are on the line. We can find the x - and y -intercepts by setting each coordinate equal to zero:

x	y
0	
	0

Substitute $x = 0$ into the equation $2x + 3y = 6$ to find the y -intercept, and substitute $y = 0$ to find the value of the x -intercept:

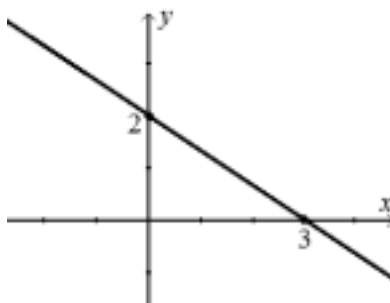
$2 \cdot x + 3 \cdot y = 6$	x	y
$2 \cdot 0 + 3y = 6 \Rightarrow y = 2 \rightarrow$	0	2
$2x + 3 \cdot 0 = 6 \Rightarrow x = 3 \rightarrow$	3	0

continued...

Chapter 2 Section 5: Linear Inequalities

Example A, continued...

We plot the intercepts we found and draw the line through them to complete Step 1.



Step 2—The region containing solutions to the inequality lies on one or the other side of the line, so now we must figure out which side of the line contains the solutions.

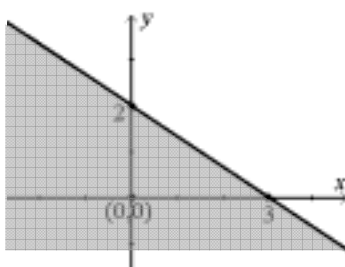
To do so, we can plug any point that's not on the line into the original inequality. A point whose coordinates satisfy the inequality is in the solution region.

The origin is a good point to use. It can help to pose Step 2 as a question: Does $(0, 0)$ satisfy the inequality?

To answer this, plug $(0, 0)$ into the inequality $2x + 3y \leq 6$, and see if the inequality is true:

$$\begin{array}{rcccl} & (0, 0) & & & \\ & \swarrow \quad \searrow & & & \\ 2 \cdot 0 & + & 3 \cdot 0 & \leq & 6 \\ & & 0 & \leq & 6 \\ & & & \text{Yes!} & \end{array}$$

It is true that $0 \leq 6$, so point $(0, 0)$ is in the region containing solutions to the inequality. We shade the region of the graph containing the solution set. Below is the graph of the solution set of the inequality $2x + 3y \leq 6$.



Note: The symbol " \leq " means "less than or equal to," so in this case the solution set includes the line, since "or equal to" means the line itself. For so-called "strict" inequalities involving " $<$ " or " $>$," the line is excluded. Exclusion of the line is usually shown by drawing a dashed line instead of a solid one, as in Example B.

Chapter 2 Section 5: Linear Inequalities

Extended Example 1a

Graph the inequality $5 - 2y \leq 6x$.

Hint: Graph the line $5 - 2y = 6x$. First make an $x|y$ table and find the x - and y -intercepts.

Step 1:

x	y
0	
	0

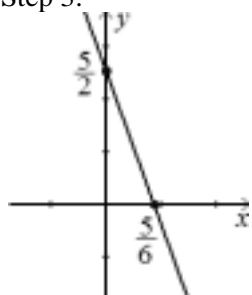
Hint: Substitute into the equation $5 - 2y = 6x$ to fill in the blanks.

Step 2:

$5 - 2 \cdot y = 6 \cdot x$	x	y
$5 - 2y = 6 \cdot 0$ $5 - 2y = 0 \Rightarrow y = \frac{5}{2}$ $5 = 2y$	0	$\frac{5}{2}$
$5 - 2 \cdot 0 = 6x$ $5 - 0 = 6x \Rightarrow x = \frac{5}{6}$ $5 = 6x$	$\frac{5}{6}$	0

Hint: Plot these points and graph the line—note that the line is solid, due to the “ \leq .”

Step 3:



Hint: Is $(0,0)$ shaded? Plug $(0,0)$ into the inequality $5 - 2y \leq 6x$.

Step 4:

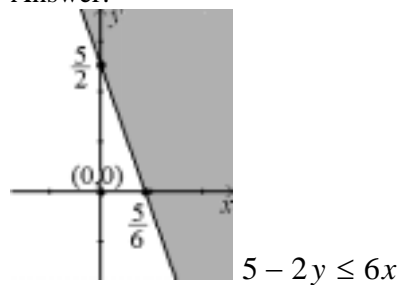
$$5 - 2 \cdot 0 \stackrel{?}{\leq} 6 \cdot 0$$

$$5 \stackrel{?}{\leq} 0$$

No!

Hint: The origin isn't in the solution set, so we know that the side of the line opposite the origin is shaded.

Answer:



Chapter 2 Section 5: Linear Inequalities

Extended Example 1b

Graph the inequality $3x - 7y \geq 21$.

Hint: Graph the line $3x - 7y = 21$. First make an $x|y$ table and find the x - and y -intercepts.

Step 1:

x	y
0	
	0

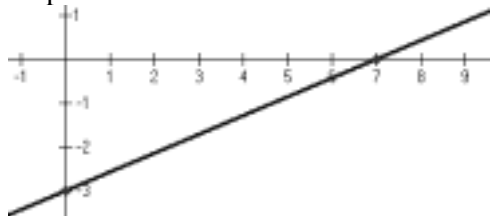
Hint: Substitute into the equation $3x - 7y = 21$ to fill in the blanks.

Step 2:

$3 \cdot x - 7 \cdot y = 21$	x	y
$3 \cdot 0 - 7y = 21 \Rightarrow y = -3$	0	-3
$3x - 7 \cdot 0 = 21 \Rightarrow x = 7$	7	0

Hint: Plot these points and graph the line—note that the line is solid, due to the “ \geq .”

Step 3:



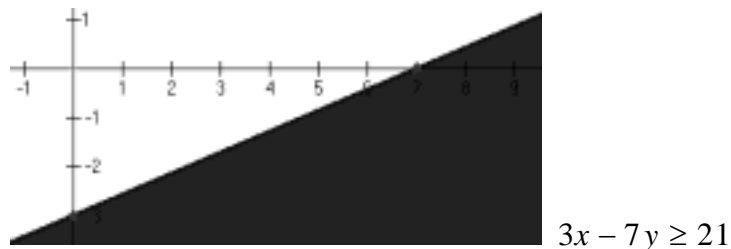
Hint: Is $(0, 0)$ shaded? Plug $(0, 0)$ into the inequality $3x - 7y \geq 21$.

Step 4:

$$\begin{aligned}
 &? \\
 3 \cdot 0 - 7 \cdot 0 &\geq 21 \\
 &? \\
 0 &\geq 21 \\
 &\text{No!}
 \end{aligned}$$

Hint: The origin isn't in the solution set, so we know that the side of the line opposite the origin is shaded.

Answer:



Chapter 2 Section 5: Linear Inequalities

Extended Example 1c

Graph the inequality $x + 2y \leq 6$.

Hint: Graph the line $x + 2y = 6$. First make an $x|y$ table and find the x - and y -intercepts.

Step 1:

x	y
0	
	0

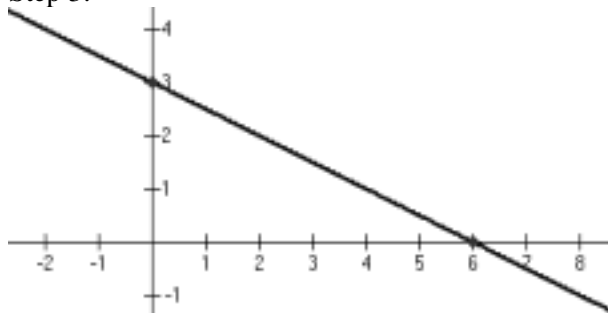
Hint: Substitute into the equation $x + 2y = 6$ to fill in the blanks.

Step 2:

$x + 2 \cdot y = 6$	x	y
$0 + 2y = 6 \Rightarrow y = 3$	0	3
$x + 2 \cdot 0 = 6 \Rightarrow x = 6$	6	0

Hint: Plot these points and graph the line—note that the line is solid, due to the “ \leq .”

Step 3:



Hint: Is $(0, 0)$ shaded? Plug $(0, 0)$ into the inequality $x + 2y \leq 6$.

Step 4:

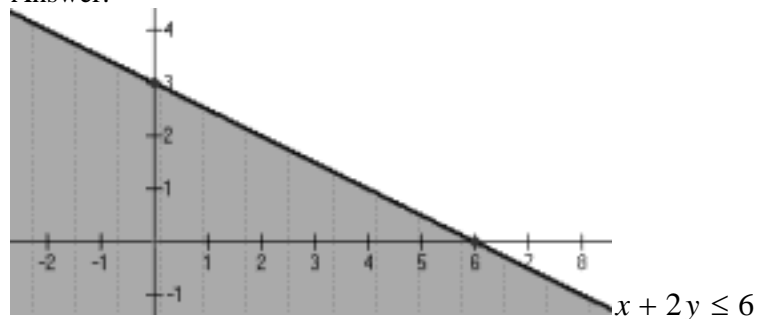
$$0 + 2 \cdot 0 \leq 6$$

$$0 \leq 6$$

Yes!

Hint: The origin is in the solution set, so we know that the origin's side of the line is shaded.

Answer:



Chapter 2 Section 5: Linear Inequalities

Example B

Graph the solution set of the inequality $y < 3x - 2$.

Step 1—Graph the line $y = 3x - 2$.

Let's find the x - and y -intercepts by setting each coordinate equal to zero.

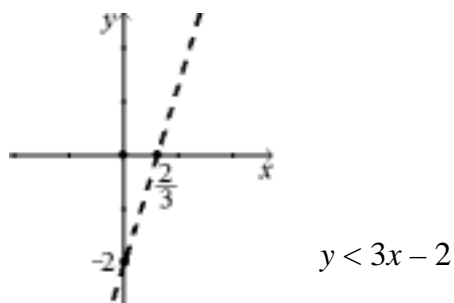
In this case, we can see that the y -intercept is -2 , so we only need to find the x -intercept:

x	y
0	-2
	0

Substituting $y = 0$ into the equation $y = 3x - 2$ to fill in the blank in our table, we get:

$y = 3 \cdot x - 2$	x	y
$y = 3 \cdot 0 - 2 \Rightarrow y = -2$	0	-2
$0 = 3 \cdot x - 2 \Rightarrow x = \frac{2}{3}$	$\frac{2}{3}$	0

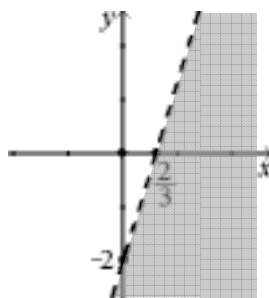
Plot these points and graph the line. This time it's a strict inequality; the line is not included, so we draw a dashed line instead of a solid one.



Step 2—Does $(0, 0)$ satisfy the inequality (in other words, is it shaded)? To answer this, just plug $(0, 0)$ into the inequality $y < 3x - 2$:

$$\begin{aligned} &? \\ 0 &< 3 \cdot 0 - 2 \\ &? \\ 0 &< -2 \quad \text{No!} \end{aligned}$$

Clearly, 0 is not less than -2 , so the origin is not part of our solution set. This means that the solutions lie on the other side of the line. Shade the solution set to complete the graph of the solution to the inequality $y < 3x - 2$:



Chapter 2 Section 5: Linear Inequalities

Extended Example 2a

Graph the inequality $x - 3y < -6$.

Hint: Graph the line $x - 3y = -6$. First make an $x|y$ table and find the x - and y -intercepts.

Step 1:

x	y
0	
	0

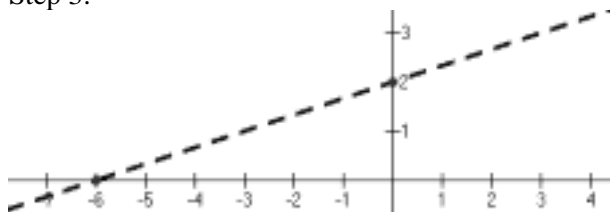
Hint: Substitute into the equation $x - 3y = -6$ to fill in the blanks.

Step 2:

$x - 3 \cdot y = -6$	x	y
$0 - 3 \cdot y = -6 \Rightarrow y = 2$	0	2
$x - 3 \cdot 0 = -6 \Rightarrow x = -6$	-6	0

Hint: Plot these points and graph the line—note that the line is dashed, due to the strict inequality, “>.”

Step 3:



Hint: Is $(0, 0)$ shaded? Plug $(0, 0)$ into the inequality $x - 3y < -6$.

Step 4:

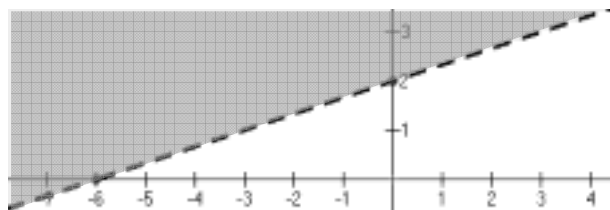
$$0 - 3 \cdot 0 < -6$$

$$0 < -6$$

No!

Hint: The origin isn't in the solution set, so we know that the side of the line opposite the origin is shaded.

Answer :



$$x - 3y < -6$$

Chapter 2 Section 5: Linear Inequalities

Extended Example 2b

Graph the inequality $5x - 6y < 30$.

Hint: Graph the line $5x - 6y = 30$. First make an $x|y$ table and find the x - and y -intercepts.

Step 1:

x	y
0	
	0

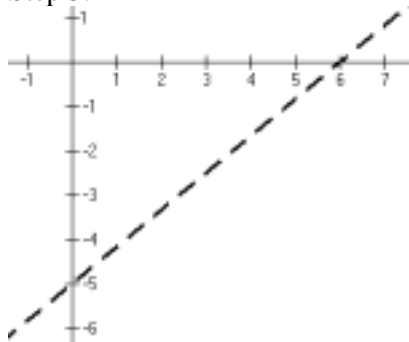
Hint: Substitute into the equation $5x - 6y = 30$ to fill in the blanks.

Step 2:

$5 \cdot x - 6 \cdot y = 30$	x	y
$5 \cdot 0 - 6 \cdot y = 30 \Rightarrow y = -5$	0	-5
$5 \cdot x - 6 \cdot 0 = 30 \Rightarrow x = 6$	6	0

Hint: Plot these points and graph the line—note that the line is dashed, due to the strict inequality, “ $<$.”

Step 3:



Hint: Is $(0, 0)$ shaded? Plug $(0, 0)$ into the inequality $5x - 6y < 30$.

Step 4:

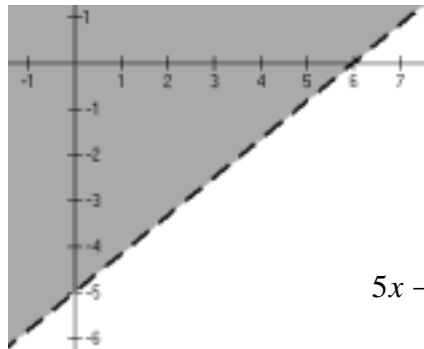
$$5 \cdot 0 - 6 \cdot 0 < 30$$

$$0 < 30$$

Yes!

Hint: The origin is in the solution set, so we know that the origin's side of the line is shaded.

Answer:



$$5x - 6y < 30$$

Chapter 2 Section 5: Linear Inequalities

Extended Example 2c

Graph the inequality $8y - 3x < 24$.

Hint: Graph the line $8y - 3x = 24$. First make an $x|y$ table and find the x - and y -intercepts.

Step 1:

x	y
0	
	0

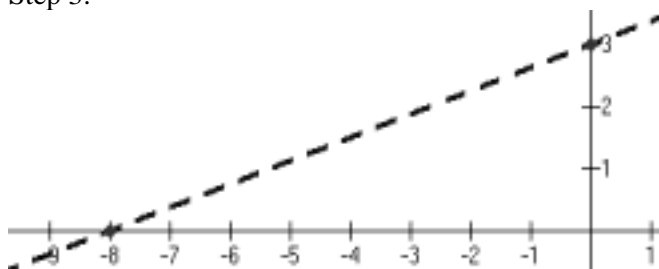
Hint: Substitute into the equation $8y - 3x = 24$ to fill in the blanks.

Step 2:

$8 \cdot y - 3 \cdot x = 24$	x	y
$8 \cdot y - 3 \cdot 0 = 24 \Rightarrow y = 3$	0	3
$8 \cdot 0 - 3 \cdot x = 24 \Rightarrow x = -8$	-8	0

Hint: Plot these points and graph the line—note that the line is dashed, due to the strict inequality, “<.”

Step 3:



Hint: Is $(0,0)$ shaded? Plug $(0,0)$ into the inequality $8y - 3x < 24$.

Step 4:

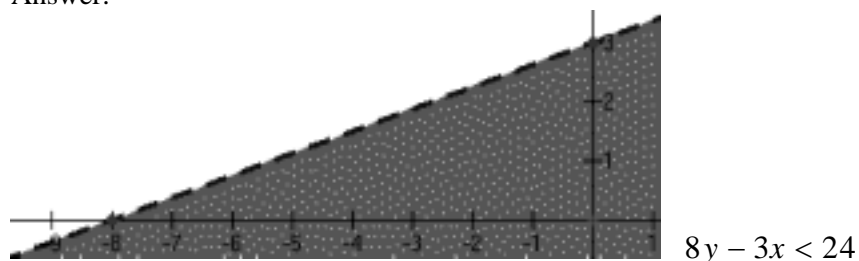
$$8 \cdot 0 - 3 \cdot 0 \stackrel{?}{<} 24$$

$$0 \stackrel{?}{<} 24$$

Yes!

Hint: The origin is in the solution set, so we know that the origin's side of the line is shaded.

Answer:



Chapter 2 Section 5: Linear Inequalities

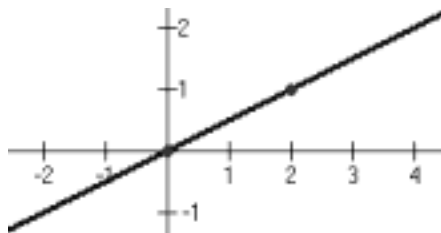
Note: We've been using the origin, $(0,0)$, as a test point in this lesson. But remember: you can use any point as a test point—as long as that point is not on the line itself. Sometimes the line passes through the origin, in such a case you can't use the origin as a test point and must use some other point.

Example C

Graph the inequality $2y \geq x$.

Step 1—Graph the (solid) line $2y = x$. Notice that for this equation, if $y = 0$ then $x = 0$, so this line goes through the origin; the x - and y -intercepts are the same point, the origin. To get a second point on the line, we'll set $y = 1$:

$2 \cdot y = x$	x	y
$2 \cdot y = 0 \Rightarrow y = 0$	0	0
$2 \cdot 1 = x \Rightarrow x = 2$	2	1

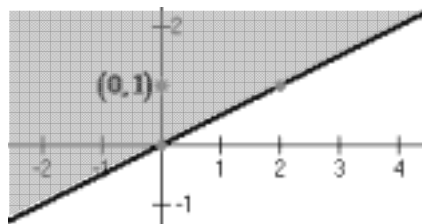


Step 2— Since this line goes through the origin, we must find another test point. So we'll ask, is $(0,1)$ shaded? To answer this, just plug $(0,1)$ into the inequality $2y \geq x$.

$$2 \cdot 1 \stackrel{?}{>} 0$$

Yes!

The answer to our Step 2 question is yes. So we need to shade the side of the line where the point $(0,1)$ lies:



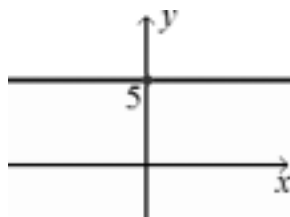
$$2y \geq x$$

Chapter 2 Section 5: Linear Inequalities

Example D

Graph the inequality $y \leq 5$.

Step 1—Graph the horizontal line, $y = 5$. There's no need to plot points since there's only one horizontal line intersecting the y -axis at 5.

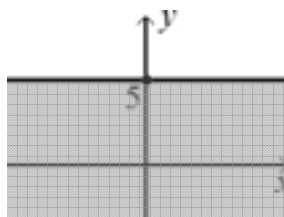


Step 2—Is $(0,0)$ shaded? To answer this, just plug $(0,0)$ into the inequality $y \leq 5$. Notice that there is no place to substitute the x -coordinate — the x -coordinate is irrelevant to this inequality.

$$\begin{array}{c} ? \\ 0 \leq 5 \end{array}$$

Yes!

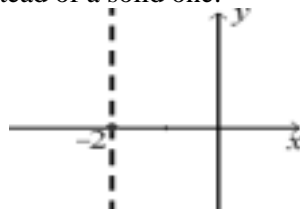
The answer to our Step 2 question is yes, so we need to shade the origin side of the horizontal line:



Example E

Graph the inequality $x > -2$.

Step 1—Graph the vertical line $x = -2$. No need to plot points this time, since there's only one vertical line intersecting the x -axis at -2 . Also, since this is a strict inequality and the line isn't included, we draw a dashed line instead of a solid one:



Step 2— Is $(0,0)$ shaded? To answer this, just plug $(0,0)$ into the inequality $x > -2$. Notice that there is no place to substitute the y -coordinate — the y -coordinate is simply not relevant to this inequality.

$$\begin{array}{c} ? \\ 0 > -2 \end{array}$$

Yes!

continued...

Chapter 2 Section 5: Linear Inequalities

Example E, continued...

The answer to our Step 2 question is yes. So we need to shade the origin side of the vertical line:



Inequalities in Slope-Intercept Form

When the inequality is written in slope-intercept form, we don't need to use a test point at all.

Just follow the simple rule:

If the inequality is of the form $y > mx + b$, then shade above the line.

If the inequality is of the form $y < mx + b$, then shade below the line.

In other words:

$$\boxed{y >} \Rightarrow y \text{ is greater than } \Rightarrow \boxed{\uparrow}$$

$$\boxed{y <} \Rightarrow y \text{ is less than } \Rightarrow \boxed{\downarrow}$$

After all, y is the vertical, up/down coordinate, and so:

If $y = mx + b$, then the point (x, y) is on the line, and

if $y > mx + b$, then the point (x, y) is above the line, and

if $y < mx + b$, then the point (x, y) is below the line.

Example F

Graph the inequality $y \geq 2x - 7$.

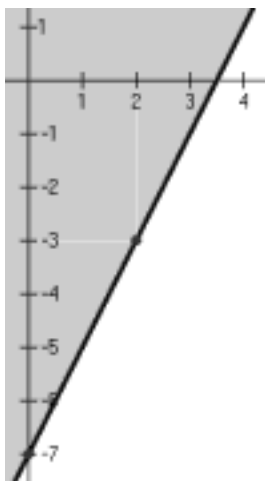
Step 1—Graph the (solid) line $y = 2x - 7$. Since this line is in slope-intercept form, we already know that the y -intercept is the point $(0, -7)$. To get a second point on the line, we'll set $x = 2$ (you could use any other value of x you wish!):

$y = 2 \cdot x - 7$	x	y
$y = 2 \cdot 0 - 7 \Rightarrow \boxed{y = -7}$	0	-7
$y = 2 \cdot 2 - 7 \Rightarrow \boxed{y = -3}$	2	-3

continued...

Chapter 2 Section 5: Linear Inequalities

Example F, continued...



Step 2— Which side to shade? Since this inequality is in slope-intercept form, and is of the form $y \geq$, “ y is greater than,” we shade above the line \uparrow :

Extended Example 3a

Graph the inequality $y \leq -\frac{2}{5}x + 3$.

Hint: To graph the line $y = -\frac{2}{5}x + 3$, start by noting the y -intercept.

Step 1:

The y -intercept is the point $(0, 3)$.

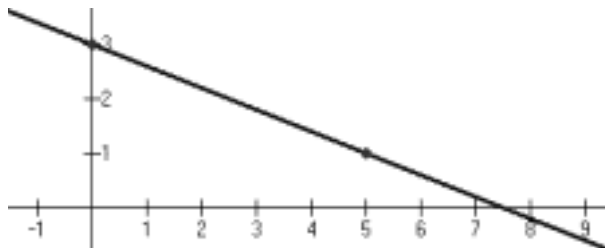
Hint: Find another point on the line $y = -\frac{2}{5}x + 3$ by letting $x = 5$. Use this point and y -intercept you found earlier to make a table of $x|y$ values.

Step 2:

$y = -\frac{2}{5} \cdot x + 3$	x	y
$y = -\frac{2}{5} \cdot 0 + 3 \Rightarrow \boxed{y = 3}$	0	3
$y = -\frac{2}{5} \cdot 5 + 3 \Rightarrow \boxed{y = 1}$	5	1

Hint: Plot these points and graph the line—note that the line is solid, due to the “ \leq .”

Step 3:



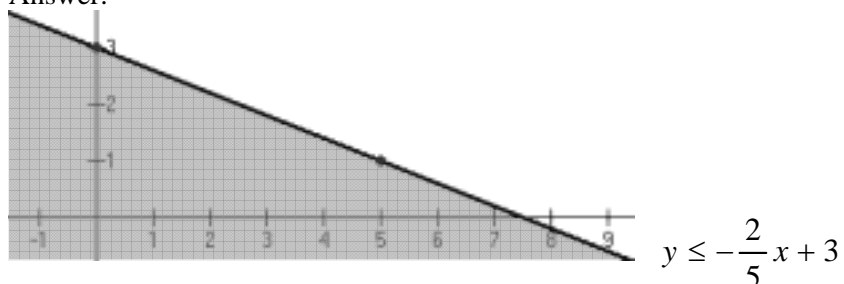
continued...

Chapter 2 Section 5: Linear Inequalities

Extended Example 3a, continued...

Hint: Since this inequality is in slope-intercept form, and is of the form $y \leq$, “y is less than,” we shade below the line \downarrow .

Answer:



Extended Example 3b

Graph the inequality $y < \frac{5}{8}x$.

Hint: To graph the line $y = \frac{5}{8}x$, start by finding the y-intercept.

Step 1:

The y-intercept is the origin, $(0, 0)$.

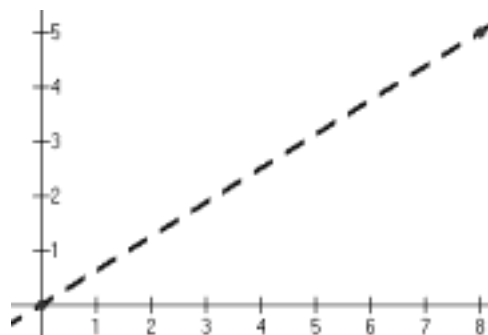
Hint: Find another point on the line $y = \frac{5}{8}x$ by letting $x = 8$. Use this point and the y-intercept you found earlier to make a table of $x|y$ values.

Step 2:

$y = \frac{5}{8} \cdot x$	x	y
$y = \frac{5}{8} \cdot 0 \Rightarrow \boxed{y = 0}$	0	0
$y = \frac{5}{8} \cdot 8 \Rightarrow \boxed{y = 5}$	8	5

Hint: Plot these points and graph the line—note that the line is dashed, due to the strict inequality “ $<$.”

Step 3:



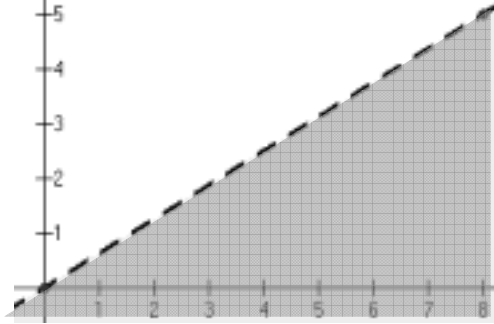
continued...

Chapter 2 Section 5: Linear Inequalities

Extended Example 3b, continued...

Hint: Since this inequality is in slope-intercept form, and is of the form $y <$, “ y is less than,” we shade below the line \downarrow .

Answer:



$$y < \frac{5}{8}x$$

Extended Example 3c

Graph the inequality $y \geq -4$.

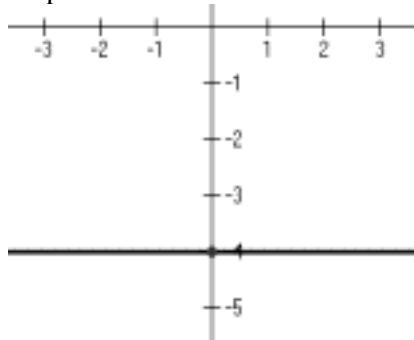
Hint: To graph the line $y = -4$, start by finding the y -intercept.

Step 1:

The y -intercept of this horizontal line is the point $(0, -4)$.

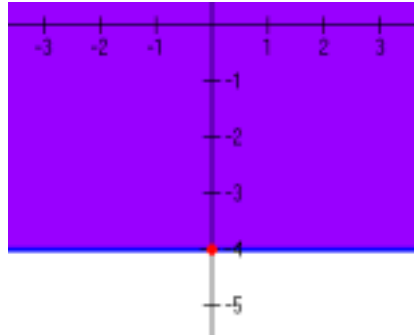
Hint: Graph this horizontal line, which is solid due to the “ \geq .”

Step 2:



Hint: Since this inequality is in slope-intercept form (since it can be written as $y \geq 0x - 4$), and is of the form $y \geq$, “ y is greater than,” we shade above the line \uparrow .

Answer:



$$y \geq -4$$

End of Lesson